Lesson 3: Graphical Representation of Uniformly Accelerated Motion in 1D

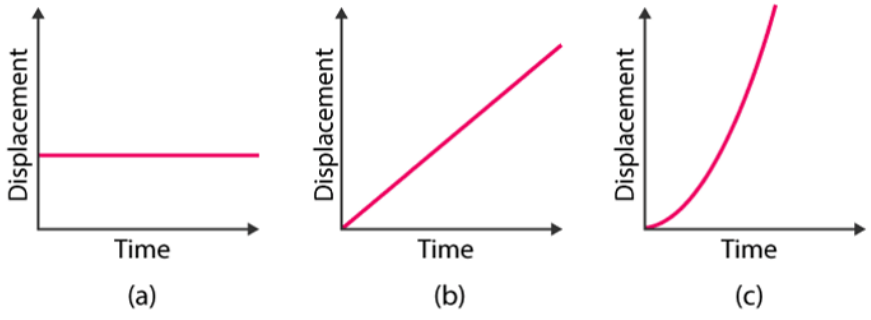
Key Terms and Concepts

- Slope of displacement-time graph give average velocity.
- Slope of velocity-time graph give the acceleration of the motion.
- The area under the velocity-time curve and the time axis gives the magnitude of the displacement covered
- Slope is equal to velocity.
- Constant velocity is explained by the straight line, while acceleration is explained by the curved lines.
- Positive slope means the motion is in the positive direction.
- Negative slope means the motion is in the negative direction.
- Zero slope means that the object is at rest

Displacement Time Graph

Displacement is defined by the distance travelled by an object from its original point. In this graph, displacement is a dependent variable and is plotted on the Y-axis while time is an independent variable and is plotted by the x-axis, it is also known as the position time graph.
The slope can be illustrated by:
S = displacement / time
$= y_2 - y_1 / x_2 - x_1$
Velocity = Δ d / Δ t
Following are three different plots for the displacement time graph:

- The first graph represents that an object is stationary for a period of time so that the slope is zero which means that the velocity of the object is zero.
- The second graph shows that the velocity of the object is constant and it increases over time so that the slope of the graph remains constant and positive.
- The third graph shows that acceleration, velocity, and displacement are constant. Slope of the curve increases along with an increase in time.



Displacement Time Graph

Given below are the key takeaways from the displacement time graph:

- Velocity of the object is equal to the slope of the curve.

- Constant velocity is expressed in a straight line, whereas acceleration is expressed as a curved line.
- When an object is at rest, it means that the slope is zero.
- The movement in a positive direction indicates a positive slope.
- Negative slope implies motion in an opposite direction.

Velocity Time Graph

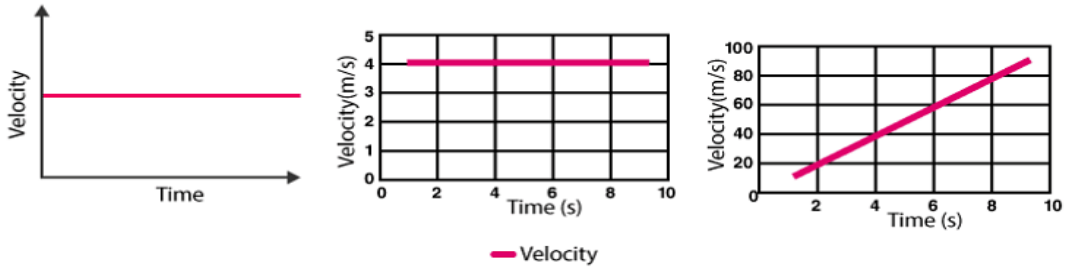
In the **velocity** time graph, velocity is the dependent variable and is represented on the Y-axis and time is an independent variable, represented on the X-axis. The slope of the velocity time graph is derived as:

Slope S = Velocity / time

$= y_2 - y_1 / x_2 - x_1$

Acceleration = Δ v / Δ t

We see that the slope of the velocity time graph is the definition of acceleration, so we can say that the slope is equal to acceleration.



Velocity Time Graph

We can understand the following points by observing the graphs:

- The steep slope of the graph represents a rapid change in velocity.
- The shallow slope represents a slow change of velocity.
- If the slope of the graph is negative, the acceleration will be negative.
- If the slope of the graph is positive, the acceleration will be positive.
- The area under the velocity shows the displacement of the object.

V-T Graph

A v-t graph is a **graphical representation** of different types of motions; the body/object under study exhibits motions by falling under a VT (velocity-time)frame.

It means that the graph will discuss the motion of the body by seeing the change in its velocity with respect to time.

The y-axis represents the velocity of the object, x-axis represents the time and the slope of the graph is nothing but **acceleration** .

While there are three main equations of motion to represent, some other important parts remain too. Velocity-time graph represents the first equation of motion, v = u + at.

The three equations of motion are:
v = u + at.
$v^2 = u^2 + 2as$. (Represented by the position-time graph)

$s = ut + \frac{1}{2}at^2$ (represented by velocity-position graph)

Various Cases of V-T Graph

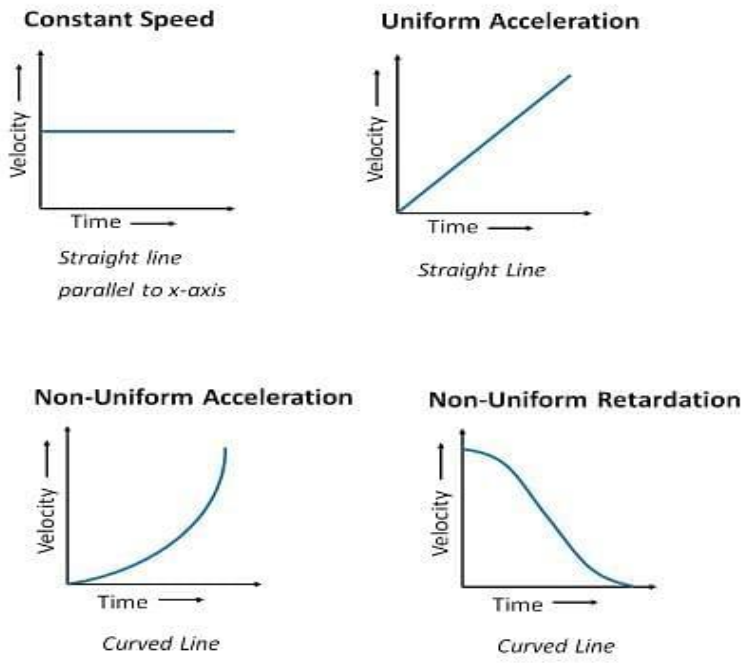
In this section we will check all the possible cases and graphical representations that we can draw with the help of a velocity-time graph.

When the object is at rest

When the object is at rest, it means there are two possibilities:

- Velocity, v = 0, or,
- Velocity, v = constant with respect to time.

Thus, the velocity-time graph will be like this:



Different Types of VT Graphs

This showcases that the object remains at rest and has no external force applied to change its state of motion.

When Object is in Motion

Once the object has been set in motion by some external force, there are mainly three cases based on the **three equations of motions** that the velocity-time graph represents:

When Object Moves with Uniform Velocity

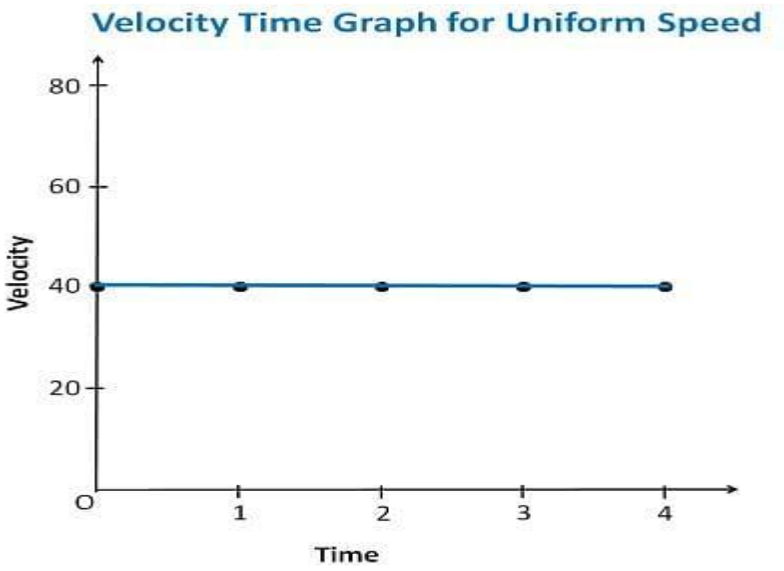
When the object moves with uniform velocity, it means that the acceleration of the body is zero. Thus, it object moves with a constant velocity throughout with slope, i.e. acceleration, a = 0.

So, from

$$v = u + at$$

v = u, means initial velocity is equal to final velocity at all times.

The velocity axis forms a straight line to coincide with the time axis.



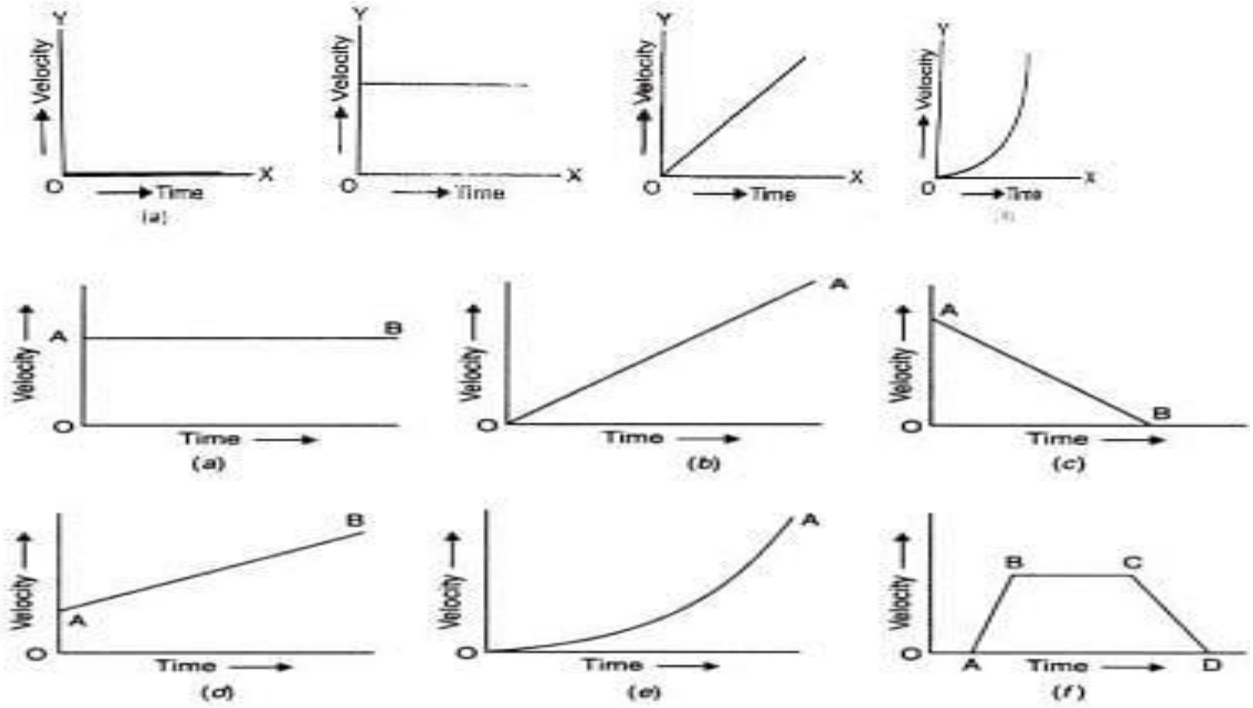
V-T Graphs with Uniform Motion

When Object moves with Uniform Acceleration

Uniform change in acceleration_ of the body means that the object changes its velocity at regular intervals of time. When object moves with uniform acceleration, there two further cases to study:

1. **When the object has positive (or upward) slope uniformly**

When the object moves with positive or upward slope constantly, it showcases positive uniform acceleration. Means, the **object accelerates_** at regular intervals in the direction of motion.



Various V-T Graphs with Different Motions

- **When the object has negative (or downward) slope uniformly**

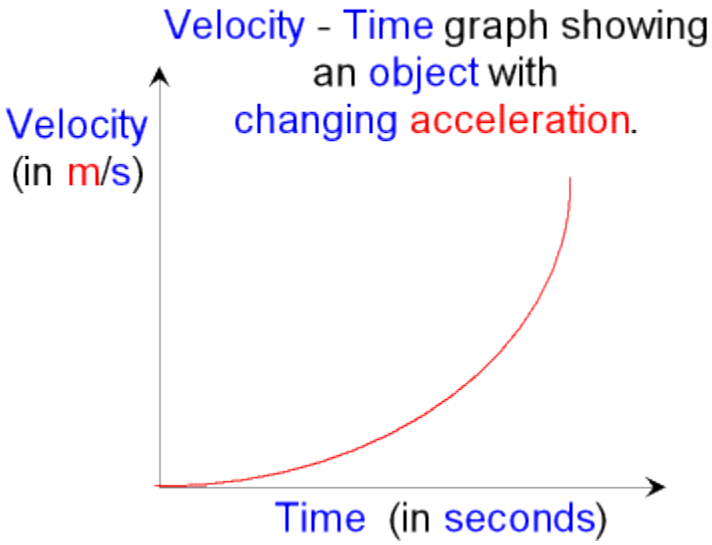
When the object moves with negative or downward slope constantly, it showcases negative uniform acceleration. Means, the object de-accelerates at regular intervals opposite to the direction of motion.

- **When object moves with changing acceleration**

When acceleration of the body keeps changing, there are two more cases possible:

- **When the object moves with positive (or increasing) acceleration**

When the body moves with positive or increasing acceleration, it is generally the case when it moves forward. For example, a car driver peddling on the accelerator of the car to make it moves ahead.



Velocity-time graphs with increasing acceleration

The magnitude of the slope continues to rise as **acceleration increases**. Also, initial velocity is often not zero in this case. Thus, by

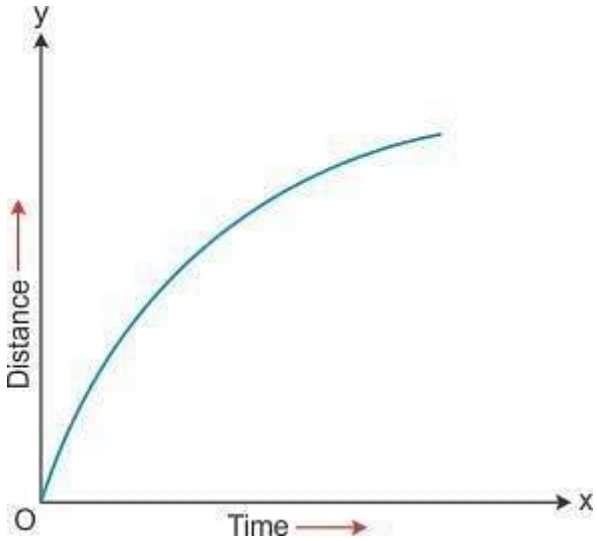
$$v = u + at$$

$$u = 0$$

So, v = at making acceleration a function of time.

- **When the object moves with negative (or decreasing) acceleration.**

When the body moves with a negative acceleration, it is often termed as 'retardation,' means decreasing acceleration. It is generally the case when it starts to come to a halt.
For example, a car driver applying brakes for the car to either slowly come to stop or slow down the speed.



Velocity Time Graph For Non-Uniform Acceleration

Acceleration Time Graph

In the **acceleration** time graph, acceleration is the dependent variable and is represented on the Y-axis and time is the independent variable and is represented on the X-axis. The slope of the acceleration time graph is as follows:

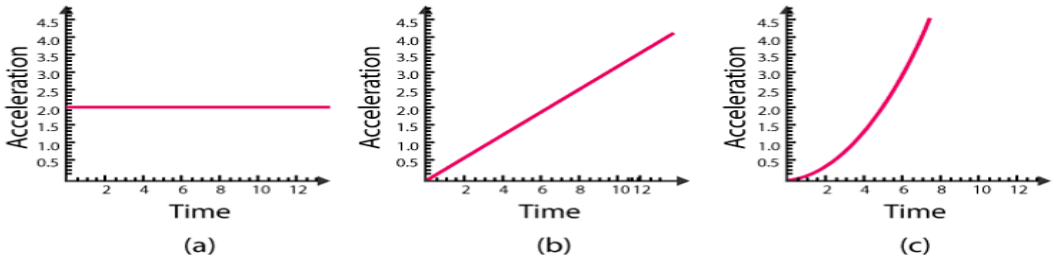
S = Acceleration / time

$= y_2 - y_1 / x_2 - x_1$

$= \Delta a / \Delta t$

The slope of the acceleration time graph is also called Jerk. Following are the points that can be concluded from the graph:

- If the slope is zero, it means that the motion has constant acceleration.
  - The area below the graph signifies the change in velocity.

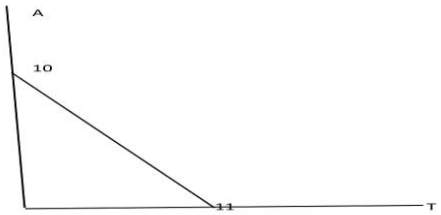


Acceleration Time Graph

Things to Remember

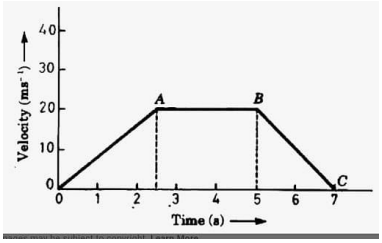
- Graphs are a way of expressing the relationship between two values.
- It is a representation of data in a two-dimensional diagram that describes the relationship between dependent and independent variables.
- In a displacement time graph, displacement is a dependent variable and is plotted on the Y-axis while time is an independent variable and is plotted by the x-axis.
- In the **velocity** time graph, velocity is the dependent variable and is represented on the Y-axis and time is an independent variable, represented on the X-axis.
- In the **acceleration** time graph, acceleration is the dependent variable and is represented on the Y-axis and time is the independent variable and is represented on the X-axis.

Example 1: When the initial velocity (v0) is 0 which starts from the rest time i.e. (t0) and the acceleration time graph is shown in the below figure, then what will be the maximum velocity?



- **Ans.** Here, the acceleration is positive till the end indicating that the velocity increases continuously but not constantly for 11 seconds. Hence, the area will represent the change in velocity (area).
- Area= ½ * 10*11 = 55
- ΔV= 55 m/s.
- Hence, maximum velocity at V0=
- = 0m/s + 55m/s
- = 55m/s^2.

Example 2: Find out the velocity of the object if it starts a v = 5m/s?



- **Ans.** In the above fig change in velocity (area)= ½*20*5+20*5+1/2*20*5= 200m/s2
- Final Velocity= Initial velocity +Change in Velocity
- = 5+200= 205 m/s^2

Things to Remember

- The most important point of the acceleration time graph is to understand the unit given in the question. Otherwise, the absolute value cannot be measured.
- The graph allows measuring the change of velocity for a given interval and not a particular period of time.
- The object can accelerate in three directions- increasing direction, decreasing direction, and change in direction.
- The acceleration of an object can change due to two reasons- one is the force and the other is the mass change.
- The slope of the Acceleration Graph represents the jerk of the graph.