Lesson 5: Uniform Circular Motion

Key Terms and Concepts

- **Uniform circular motion** is a circular motion with constant speed but continuously changing direction.
- **Angular displacement** is the angle swept by the rotating object.
- **Tangential displacement** the tangential distance on the circumference of the circle
- **Centripetal acceleration** is acceleration in circular motion caused by the change in the direction of velocity. Its direction is always towards the center of the circle.
- **Centripetal acceleration** is acceleration in circular motion caused by the change in the direction of velocity. Its direction is always towards the center of the circle.
- **Centripetal force** is a force that causes centripetal acceleration. It is always perpendicular to the direction of motion of the object.

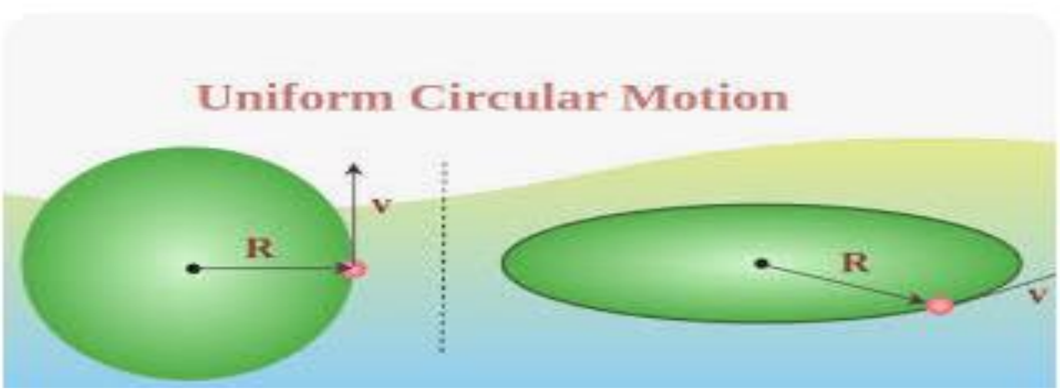
Uniform Circular Motion

- Uniform circular motion is a special type of curvilinear motion in which an object travels in a circle with a constant speed but its direction of motion continuously changes.
- A mass attached to the end of a string and moves in a horizontal circle, the second, minute, and hour hands of a watch, the motion of a turntable, and any point on a propeller spinning at a constant rate are examples that could demonstrate uniform circular motion.

Examples of Uniform Circular Motion

The examples of uniform circular motion are as follows –

- The motion of artificial satellites around the planet exemplifies uniform circular motion, which is kept in a circular orbit around the planet by the gravitational attraction of the earth.
- The moon revolves around Earth in a circular motion.
- The movement of electrons around the nucleus of an atom is a uniform circular motion.
- The rotation of the windmill blades.
- A watch with a circular dial has a uniform circular motion at the tip of the second hand
- Uniform circular motion, therefore, occurs when a body moves in a circular path at a consistent speed



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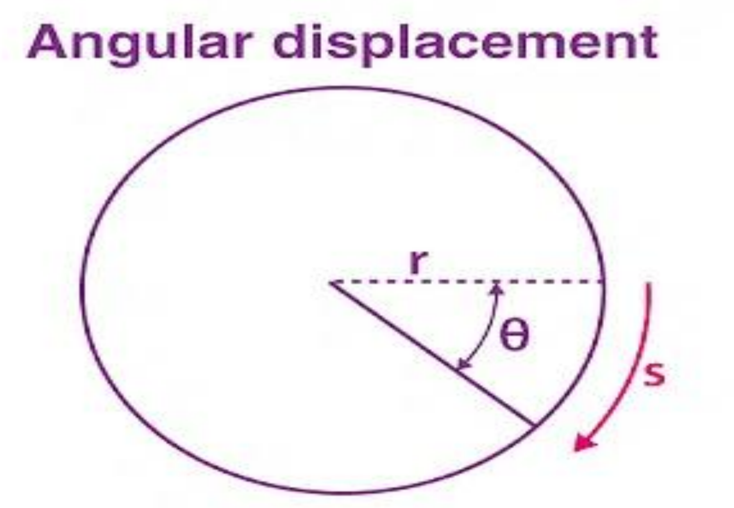
Things to Remember

- A circular motion is defined as a body movement that follows a circular route.
- The motion of a body going at a constant speed along a circular path is known as uniform circular motion.
- When a body moves in a circle at a constant speed, the work it does is zero. This implies that the centripetal force does not work.
- The moving object is accelerated by a centripetal force in the direction of the center of rotation because the object's velocity vector is in a continually changing direction.
- The frictional force between the tires and the ground provides the essential centripetal force for turning cars on the roads.
- The motion of artificial satellites around the planet exemplifies uniform circular motion.

Angular and tangential displacement

Angular Displacement

- **Angular Displacement** is the **angle** through which a line or point rotates about a specific axis.
- It is the angle formed when an object moves in a circular motion.
- For example, a pole dancer spinning on a pole makes 360o or 180o.
- Therefore, π or 2π will be the angular displacement of the pole dancer.
- It is measured in degrees or radians and is denoted by the Greek letter theta θ. Mathematically,
- **Angular displacement = θ = s/r**
- Where r is the radius of the **circle** and s is the distance covered by the body.
- The unit of angular displacement in the SI system is radian.
- Since angular displacement is the ratio of these two quantities hence it is dimensionless.
- (s) On the other hand, tangential displacement is the distance covered by the rotating object along the curved path (circumference of a circle
- 

Therefore,

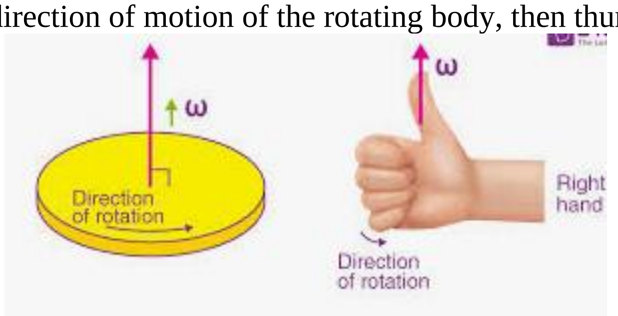
$\boldsymbol{\theta = \frac{S}{r} = \frac{2\pi r}{r} = 2\pi\ radian}$

Angular velocity and tangential velocity

Angular Velocity

Angular velocity is a **vector number that indicates the angular speed or rotational speed of an object** and the axis around which the object is revolving.

- **The amount of change in the particle's angular displacement over time is known as angular velocity.**
- The angular velocity vector's course is vertical to the plane of rotation, as indicated by the right-hand rule.
- **An example of angular velocity is a roulette ball on a roulette wheel**, a race car on a circular circuit, or a Ferris wheel.
- Furthermore, **the object's angular velocity is its angular displacement with respect to time**.
- Additionally, as an object moves along a circular path, the central angle that corresponds to the object's position on the circle shifts.
- Furthermore, the angular velocity, denoted by the symbol **ω,** is the rate at which this angle changes with respect to time.
- It is denoted by Greek letter omega (**ω**) and its standard unit is radians per second
- $\boldsymbol{\omega = \frac{\Delta\theta}{r}}$
- Angular velocity is a vector quantity. The direction of angular velocity is determined by right hand rule.
- According to right hand rule, if you hold the axis with your right hand and rotate the fingers in the direction of motion of the rotating body, then thumb will point the direction of the angular velocity.



- A tangent is a line that touches a curve at a single point. As the name suggests,
- Tangential velocity describes the motion of an object along the edge of the circle whose direction at any given point on the circle is always along the tangent to that point. It is the linear component of angular velocity at any point along the circular path
- Tangential velocity is equal to the tangential distance divided by time
- $\mathbf{V_t} = \boldsymbol{\frac{S}{r}}$
- After one complete rotation, the tangential displacement is equal to the circumference of the circle, and time taken for one complete circle is called the period, T. With this the equation for tangential velocity will be:
- $\mathbf{V_t} = \boldsymbol{\frac{2\pi r}{T}}$
- The SI unit of tangential velocity is meters per second.

The relationship between angular and tangential velocity

- Consider the formula of angular velocity, that is,
- $\omega = \Delta\theta / \Delta t$
- Multiplying both sides by radius r, we get,
- $r\ \omega = r\ \Delta\theta / \Delta t$
- We know the term $r.\Delta\theta$ is the distance an object travels in a circular path of radius r. Therefore, the equation becomes,
- $\Delta s / \Delta t = r\ \omega$
- The term on the right-hand side of the above equation denotes the ratio of distance and time, which is velocity. Therefore,
- **$v = r\ \omega$**
- $v = \Delta s / \Delta t$
- Therefore, linear velocity equals the product of radius and angular velocity.
- **Note that**:- angular velocity is the same for all points on the rotating object, while tangential velocity depends directly on its distance from the axis of rotation.

Examples 5.1:-**Determine the linear velocity of a point rotating at an angular velocity of 16 π radians per second at a distance of 8 cm from the center of the rotating object.**

Given

- Angular velocity, $\omega = 16\ \pi$ rad/s
- Radius of the circular path, r = 8 cm = 0.08 m

The relation between angular velocity (ω) and linear velocity (v) is given

$= r\omega$

$\Rightarrow v = 0.08 \times 16\ \pi = 4.01$ m/s

Things to Remember

- $S = r\theta$ simplifies the computation by putting the value of in radians in the equation.
- The angular velocity, like the displacement, is an axial vector quantity. Using the Right-hand Thumb Rule, we can determine its direction. The following rule applies:
- Curl your fingers counterclockwise, and the direction of angular motion is the thumb pointing outwards (along the axis). Similarly, curling your fingers clockwise gives you the direction of your thumb pointing inwards.
- This equation holds for any particle with a rigid body at any given time.
- The speed (velocity isn't really right here; that's a vector) will increase if the radius is changed by gently reducing the string. Again, when the radius decreases, velocity increases, but in order to stay in orbit, one must fire engines to accelerate the satellite!

Example:- Determine the angular velocity if 7.2 revolutions are completed in 5 seconds.

Given

- Total number of revolutions, n = 7.2
- Time taken to complete 7.2 revolutions, t = 5 seconds
- The frequency of the revolution is given by $f = n/t$
- $\Rightarrow f = 7.2/5 = 1.44$ Hz
- Angular velocity is given b
- $\omega = 2\pi f$
- $\Rightarrow \omega = 2 \times 3.14 \times 1.44 = 9.04$ rad/s

Centripetal Acceleration: Acceleration in Uniform Circular Motion

- We know from kinematics that acceleration is caused by a change in velocity, either in magnitude or in direction or both. Therefore, an object undergoing uniform circular motion is always accelerating, because it always changes it direction of motion.
- We call the acceleration of an object moving in uniform circular motion the centripetal acceleration, a_c . Centripetal means center seeking or a towards the center.
- Centripetal acceleration is in the direction of the change in velocity, which points directly toward the center of the circular path.
- Because a_c is directed towards the center of a curved path along the radius it is also called radial acceleration. This direction is shown with the vector diagram in the Figure.
- Centripetal acceleration is always directed toward the center of the circular in which the body is moving and it is perpendicular to the direction of tangential velocity.
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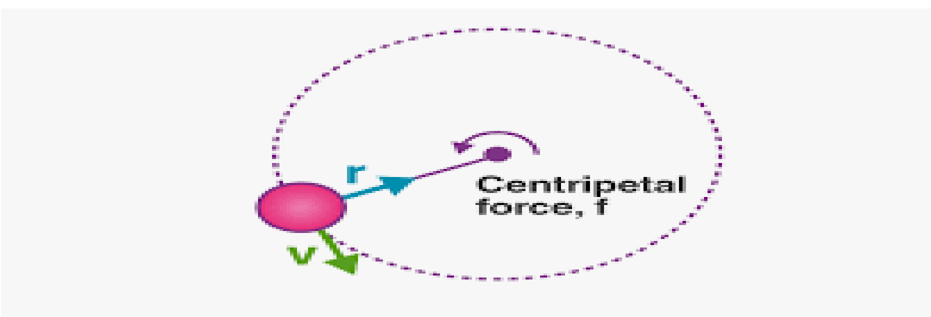
    - 

Centripetal Acceleration

- $\mathbf{a_c = \frac{V2}{r}}$
- Since, in terms of angular velocity, the equation for tangential velocity is: v= ωr,
- **a= ω² r** → $\mathbf{a = \omega^2 r}$
- The SI unit of centripetal acceleration is m/s^2

. **Examples 5.3:-A rock tied to a string is moving at a fixed speed of 20.0 m/s in a circle having a radius of 6.0 m. Evaluate the approximate magnitude of the centripetal acceleration of the rock**

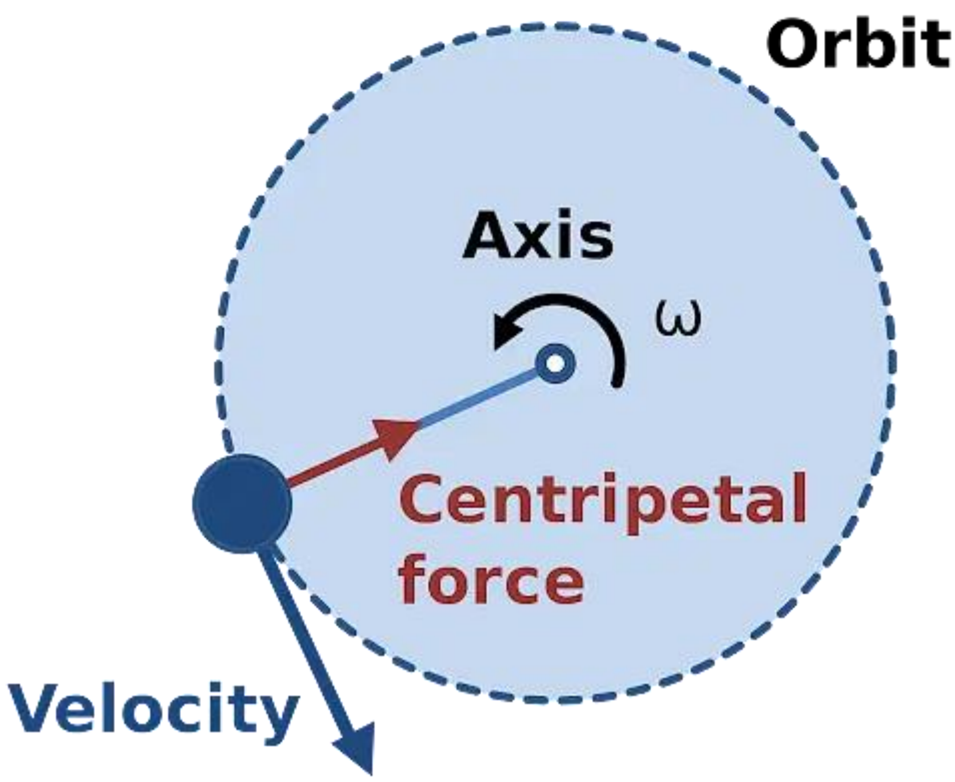
- **Given,**
- the velocity of the rock tied to the string, v = 20 m/s
- the radius of the circular path, r = 6 m
- From the formula of centripetal acceleration,
- $a_c = v^2 / r$
- $a_c = (20)^2/ 6 = 66.67\ m/s^2$
- Therefore, centripetal acceleration is $66.67\ m/s^2$

Examples 5.4:- In the case of a slot-car set, its maximum centripetal acceleration without being ejected from its track is noted to be 3.8 meters per second squared. It is noticed that these slot cars fly off their track when they exceed 1.1 meters per second. What is the radius of the curve in the track? Answer in meters

- **Ans.** Here, Maximum possible centripetal acceleration a = 3.8 m/s^2
- And, the maximum speed which can be attained by these particular cars without flying off its track v is 1.1 m/s.
- Using the centripetal acceleration formula:
- $a_c = v^2 / r$
- $r = v^2 / a_c = (1.1\ m/s)^2 / 3.8\ m/s^2 = 0.32m$
- Therefore, the Radius of the curve is 0.32m

Centripetal Force

- The force which is acting on an object in curvilinear motion and which is discovered to be oriented towards the axis of rotation or center of curvature.
- The unit of centripetal force is the newton.
- In Newton, we can represent centripetal force. But first, let's talk about how the object ended up on the circular path in the first place. Newton's first law states that unless moved on by an external force, an item will continue to move in a straight line. The centripetal force is the external force at work here.
- Centripetal force is the net force that causes an object's centripetal acceleration in a circular motion. The centripetal force is directed toward the center, which is perpendicular to the body's motion.



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- Calculating Centripetal Force
- **$F = mv^2/r$**
- **Here, F is the centripetal force,**
- **m is the object's mass,**
- **v is the object's speed or velocity, and**
- **r is the radius.**

Real-Time Examples of Centripetal Force

- **This power can be felt in a variety of scenarios in real life. A few of them are listed below.**
- Turning a car (or any vehicle)
- Riding a roller coaster and going through loops
- Using a string to spin a ball
- Planets in the solar system orbit the sun in a heliocentric orbit.

Examples 5.5: A 23 kg girl is riding a merry-go-round with a radius of 4 m. What is the centripetal force on the girl if her velocity is 5 m/s?

Given,

- mass of the girl, m = 23 kg
- the radius of the circular path r= 4 m
- the tangential velocity of the girl, v = 5 m/s
- The centripetal force acting on the girl is given by
- $F_c = mv^2/r$
- $F_c = (23 \times 5^2) / 4 = 143.75$ N

Uniform Circular Motion in a Horizontal Plane

- Horizontal circular motion indicates circular motion across a horizontal surface. There are various day-to-day examples such as the motion of a mass attached to the end of a string and made to rotate in a horizontal circle, In this case the centripetal force is provided by the tension (T) in the string. Hence,

  $$\mathbf{T=}\frac{\boldsymbol{mV2}}{\boldsymbol{r}}$$

The conical pendulum

Suppose a particle of mass m is tied to a string of length **l** and then whirled round in a horizontal circle of radius r, with O fixed directly above the center B of the circle, as shown in the Figure .

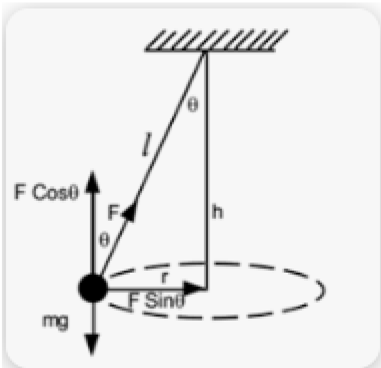
If the circular speed of the particle is constant, the string turns at a constant angle to the vertical. This is called a conical pendulum

The horizontal component of the tension in the string (T), points towards the center of the circle, hence it provides the centripetal force

$$T\sin\Theta=\frac{\boldsymbol{mV2}}{\boldsymbol{r}}$$

- The weight (mg) of the particle is balanced by the vertical component of the tension in the string.
- $T \cos\theta = mg$
- Dividing the above equation by the lower one gives:

$$\tan\Theta = \frac{\boldsymbol{V2}}{\boldsymbol{rg}}$$



Motion of a car Round a Banked Road

Suppose a car is moving round a banked road in a horizontal circular path of radius, r, as shown in the Figure If the only forces at the wheels are the normal reaction forces FN, that is there is no side-slip or strain at the wheels, the force towards the center of the track is

N sin θ=$\frac{mV2}{r}$