Lesson 4:The first Condition of Equilibrium.

Static Equilibrium

The static equilibrium of a particle or any system is defined as the condition when it is completely at rest i.e. no translational or rotational motion is observed.

- The term 'static' means that the body is completely at rest, whereas 'equilibrium' means that all of the forces acting on the system are balanced.
- The nature of the forces is said to be completely balanced only when the forces that are directed upward are balanced and their effect is canceled out by the forces that are directed downward are the forces.
- Similarly, forces directed to the left must be balanced by forces directed to the right.
- This does not imply that the forces are identical in magnitude, but that they are completely balanced and hence cancel each other out.

Conditions of Equilibrium

To be in a state of equilibrium, a system must not undergo any kind of linear or rotational acceleration. That simply means that the net acting force, **torque**, or both must total to zero.
The two conditions of equilibrium are as follows

First Condition of Equilibrium

The first condition of equilibrium is also known as the condition for translational equilibrium.
A rigid body is said to be in translational equilibrium if it remains at rest or moving with constant velocity.
Let F_1, F_2, F_3 ... be the external forces acting on a rigid body of mass m, then according to **Newton's second law of motion**,
If velocity v is constant, then $\sum F_{ext} = 0$

Thus a body is said to be in translational equilibrium if the net external force acting on the body is zero i.e.

$$\sum \mathbf{F_{ext}} = \mathbf{0}$$

Thus a body is said to be in translational equilibrium, if its total linear momentum remains constant i.e. does not change with time i.e.

$$\mathbf{p = constant}$$

The image below shows a stationary person in static equilibrium. As the image shows, the forces acting on him sum up to zero.

The car in the image below is in dynamic equilibrium since it moves at a constant velocity. There are horizontal and vertical forces, but there is no net external force in any direction.

Second Condition of Equilibrium

The first condition of equilibrium is also known as the condition for rotational equilibrium.

A rigid body is said to be in rotational equilibrium if the body does not rotate with or rotate with constant angular velocity.

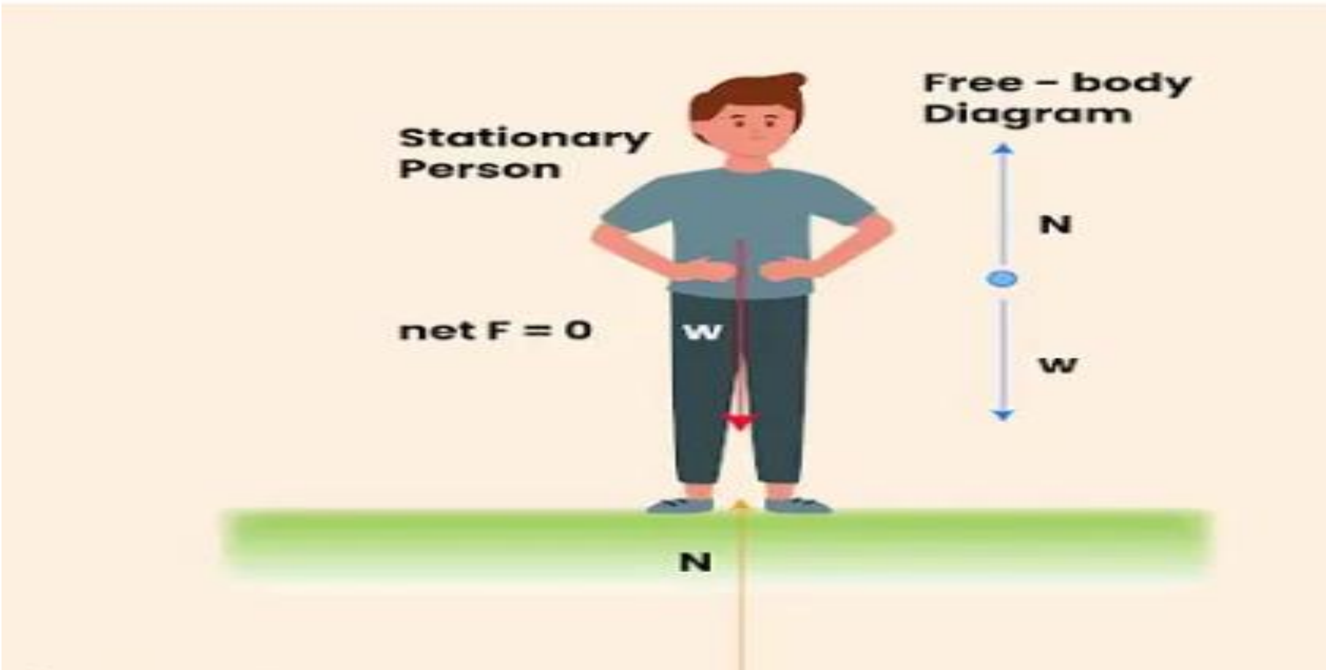
Let τ_1, τ_2, τ_3, ……… be the external torque acting on a rigid body, then net external torque is given by

$\sum \tau_{ext} = \tau_1 + \tau_2 + \tau_3 + ...$

Translational Equilibrium of an Object

The condition of translational equilibrium, as per Newton's second law, states that the object experiences zero net force.

- In mathematical terms, this condition can be expressed as F = 0, where the sum of all forces in all three dimensions (Fx, Fy, Fz) must be equal to zero. (Fx = 0, Fy = 0, and Fz = 0)
- To achieve static equilibrium, it is essential to thoroughly assess both the magnitude and direction of forces
- It is crucial to satisfy the condition of static equilibrium in all dimensions to ensure balance and stability.
- In the examples discussed, the focus is on objects moving within the x-y plane, which implies a two-dimensional analysis.
- The condition of static equilibrium applies to forces acting in all three dimensions (x, y, and z axes) to ensure the object remains in a state of rest or uniform motion



Examples 4.1:-A traffic light weighing 100 N hangs from a vertical cable tied to two other cables that are fastened to a support, as in Figure 4.33a. The upper cables make angles of 37° and 53° with the horizontal. Find the tension in each of the three cables

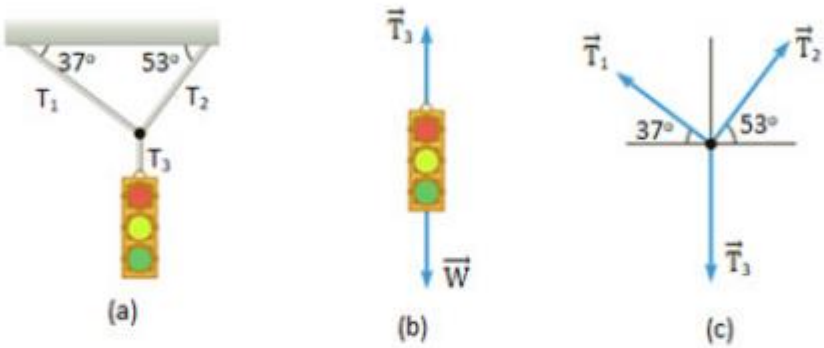


Figure 4.33 A traffic light suspended from two strings Solution

Solution

Drawing a free-body diagram is highly helpful in solving problems involving Newton's laws. As shown in the free-body diagram, the traffic light is under the action of three tension forces, $\vec{T}_1$, $\vec{T}_2$ and $\vec{T}_3$. For it is in equilibrium, we write

$$\sum \vec{F} = 0$$

$$\vec{T}_1 + \vec{T}_2 + \vec{T}_3 = 0$$

But from Figure b, $\vec{T}_3 = \vec{W}$ and we have $\vec{T}_1 + \vec{T}_2 + \vec{W} = 0$

Breaking the forces into their components and applying the first condition of equilibrium we have

$$\sum \vec{F}_x = 0, T_{2x} - T_{1x} = 0 \quad T_2 \cos 53^\circ - T_1 \cos 37^\circ = 0 \quad 0.6T_2 - 0.8T_1 = 0$$

$$\sum \vec{F}_y = 0, T_{2y} - T_{1y} - W = 0 \quad T_2 \sin 53^\circ - T_2 \sin 37^\circ = W \quad 0.8T_2 - 0.6T_1 = 100N$$

Solving for T_1 *and* T_2 *we get* $T_1 = 60N$ *and* $T_2 = 80N$

Examples 4.2:-Mass m= 10kg is supported by two strings is in equilibrium as shown in Figure 4.34. Find the tension in the horizontal string. Take θ = 53o

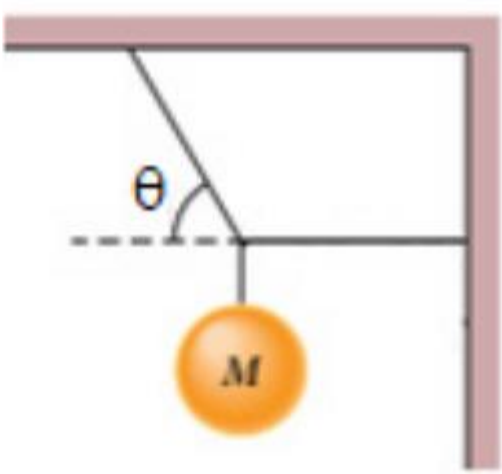
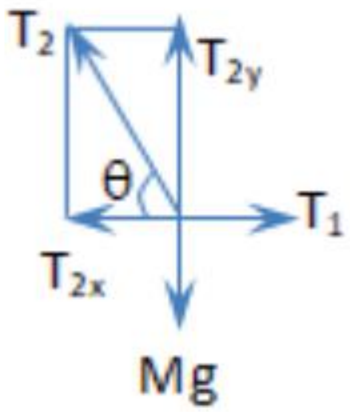
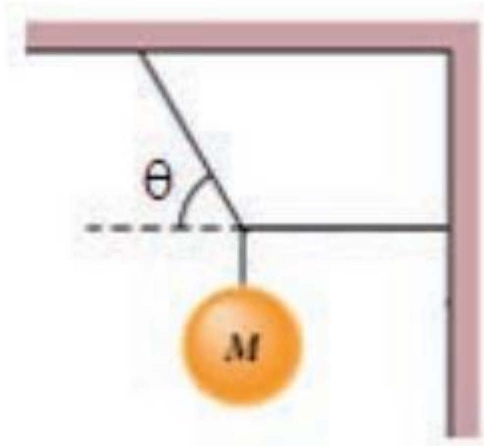


Figure 4.34 A mass in equilibrium

Solution





$T_1 + T_2 + W = 0$

$T_{1x} - T_{2x} = 0$, $T_1 -- T_2 \cos 53^o = 0$, $T_1 -- 0.6\ T_2 = 0$, $T_1 = 0.6\ T_2$

$T_{ly} - W = 0$, $T_1 \sin 53^o = Mg$, $0.8\ T_1 = Mg$,
$T_1 = Mg/0.8 = 1.25\ Mg = 1.25 \text{ x } 10 \text{ kg x } 10 \text{ m/s}^2 = 125 \text{ N}$