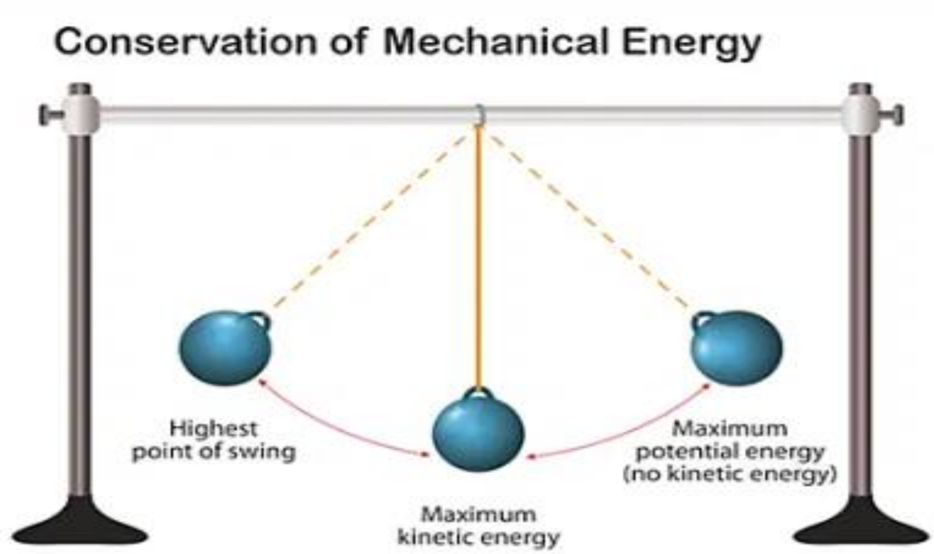
Lesson 6: Conservation of mechanical energy

Key Terms and Concepts

- **Mechanical Energy** is the sum of kinetic and potential energy in an object employed to perform a specific task.
- **Kinetic energy:** Energy due to motion
- **Potential energy:** Energy due to position or shape

Conservation of Mechanical Energy:

- **The principle of conservation of mechanical energy is that energy can neither be created nor be destroyed but it can only be transformed from one form to the other**.
- Mechanical energy refers to the sum of **potential energy** and **kinetic energy** in an object that is used to do a particular work.
- The principle of conservation of mechanical energy states that if an isolated subject in a system is subjected to **conservative forces** only, then the mechanical energy remains constant.
- The potential energy of the system is highest when the bob reaches its maximum height, whereas the kinetic energy is zero.
- The kinetic energy is the largest at the mean position, whereas the potential energy is zero.
- The system has both kinetic and potential energy, the sum of which is constant, between these two extremes.
- The law of conservation of mechanical energy states that in the absence of dissipative forces like air resistance and friction, the total mechanical energy of an object or system of objects remains unchanged.
- $\Delta ME = 0$
- $\Delta KE + \Delta PE = 0$
- (KE KEi) + (PE PE) = 0
- $\mathbf{KE_i + PE_i = KE_f + PE_f}$



Example 6.1:-

A 10 kg block is released from rest at the top of a smooth inclined plane 10 m in length, as shown in Figure 4.48. Find the speed of the block as it reaches the bottom of the inclined plane.

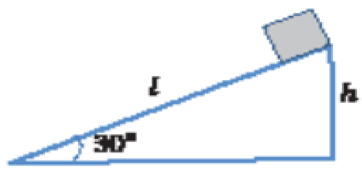


Figure 4.48

Solution

There is no friction between the block and the inclined surface. Applying the law of conservation of mechanical energy we write

$$\Delta KE + \Delta PE = 0$$

$$KE_i + PE_i = KE_f + PE_f$$

The block starts from rest so that $KE_i = 0$, and at the bottom of the incline $PE_f = 0$

$$PE_i = KE_f$$

$$mgh = \tfrac{1}{2} mv^2$$

$$v = \sqrt{2gh},\ h = l \sin 30° = (10m)(0.5) = 5m$$

$$Therefore\ v = \sqrt{2(9.8m/s^2(5m)} = 9.9m/s$$

Example 6.2:- For a baseball field, the distance between home plate and the center field wall is about 123m and the wall is 5m tall. If a player hits the ball at a level of 1m off the ground, an angle of 55° above the horizontal, and a velocity of 40m/s, what is the total velocity of the ball as it passes over the center field wall at a height of 33m? Neglect air resistance and assume g=10ms2

Method 1: Conservation of Energy

Note: The only reason we can use this method is because we know the height at which the ball crosses over the center field wall. Without that height, we would have to do method 2.

We will first split the initial velocity of the ball into its components:

$$v_{xi} = 40cos(55^{\circ}) = 22.9\frac{m}{s}$$

$$v_{yi} = 40sin(55^{\circ}) = 32.8\frac{m}{s}$$

Since we are neglecting air resistance, the x-component stays constant. Therefore we can say:

$$v_{xf} = 22.9\frac{m}{s}$$

We can find the final y-componenet using our conservation of energy equation:

$$E = U_i + K_i = U_f + K_f$$

Substituting in our expressions:

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

Canceling out mass and rearranging for final velocity:

$$g(h_i - h_f) + \frac{1}{2}v_i^2 = \frac{1}{2}v_f^2$$

$$v_f = \sqrt{2g(h_i - h_f) + v_i^2}$$

Note that this is one of the big five kineamtics equations with which you should be familiar. Plugging in our values, we get:

$$v_f = \sqrt{2(10\frac{m}{s^2})(1m - 33m) + (32.8\frac{m}{s})^2} = \sqrt{-640\frac{m^2}{s^2} + 1073.616\frac{m^2}{s^2}}$$

$$v_{fy} = 20.8\frac{m}{s}$$

Now that we have the components, we can combine them to get the final velocity:

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(22.9)^2 + (20.8)^2}$$

$$v_f = 31.0\frac{m}{s}$$