

Lesson 3: Algebraic Method of addition of vectors in Two Dimensions (2-D) and Scalar multiplication

- Scalar quantities are fully described by their magnitude, allowing for simple arithmetic operations such as addition and subtraction.
- In contrast, vector addition requires considering both magnitude and direction.
- When adding collinear vectors, such as force vectors F_1 and F_2 acting in the same direction, the resultant vector R has a magnitude equal to the sum of the magnitudes of F_1 and F_2 , and it is directed along the same line.
- If the vectors are in opposite directions, the resultant vector R will have a magnitude equal to the absolute value of the difference in magnitudes and will be directed along the larger vector.
- For perpendicular vectors, the resultant is found using the Pythagorean Theorem for magnitude and trigonometry for direction.
- Adding vectors in two dimensions often involves graphical methods and trigonometry, which may lack precision.
- A more general and accurate approach is the component method, where vectors are resolved into their x and y components using the Cartesian coordinate system.
- For a vector A , its x-component A_x is $A\cos(\theta)$ and its y-component A_y is $A\sin(\theta)$, where θ is the angle made with the positive x-axis.
- The process of breaking a vector into its components is known as vector resolution.
- The resultant of two or more vectors is found by summing their respective x and y components.
- The magnitude of the resultant vector is determined using the Pythagorean theorem, and its direction is found using the tangent function to calculate the angle with the x-axis.
- There are two different kinds of products of vectors: the scalar (dot) product and the vector (cross) product.
- The scalar product of two vectors results in a scalar quantity, while the vector product results in another vector.
- When a vector is multiplied by a scalar, the magnitude of the vector changes according to the magnitude of the scalar, but the direction remains the same.
- If the scalar is positive, the new vector remains in the same direction as the original;
- if the scalar is negative, the new vector points in the opposite direction.
- This process, known as scalar multiplication, changes the size of the vector but not its direction.
- The scalar product of two vectors is denoted by $A \cdot B$ and is also called the dot product.
- The scalar product of two vectors $\mathbf{a}$ and $\mathbf{b}$ with magnitudes $|\mathbf{a}|$ and $|\mathbf{b}|$ is given by $|\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between the vectors.
- This can be expressed as $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where $|\mathbf{a}|$ and $|\mathbf{b}|$ are the magnitudes of vectors $\mathbf{a}$ and $\mathbf{b}$, and $\cos\theta$ is the cosine of the angle between them.
- This product results in a scalar value, which represents the product of the magnitudes of the two vectors and the cosine of the angle between them.
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