Lesson 1: Relations

Lesson Objective:

Dear learner, at the end of this lesson you will be able to:

- define relations
- write domain and range of a relation in interval notation, inequalities and set notation

Brainstorming Activities:

Dear learner can you give the correct answer for the following activity?

Let A= {1,2,3} and B = {

Definition:

- **Given two sets A and B, any set $R = \{(x, y) : x \in A \text{ and } y \in B\}$, is called a relation from set A to set B**

Example: 1

1. Let $A = \{1,3,5,7\}$ and, $B = \{6,8\}$ and R be the relation "less than" from A to B. Then, $R = \{(1,6),(1,8),((3,6),(3,8),(5,6),(5,8),(7,8)\}$.
2. Let $A = \{1,2,3,4,5\}$ and, $B = \{a,b,c\}$ .The following are relations from $A to\ B$
   i) $R_1 = \{(1,a)\}$
   ii) $R_2 = \{(2,b),(3,b),(4,c),(5,a)\}$
   iii) $R_3 = \{(1,a),((2,b),(3,c)\}$

Note: A relation is the set of ordered pairs

1.1.Domain and Range of relations

Given a relation R from set A to B;

Domain of R: the set of all the first components of the elements of R.

$\boldsymbol{Dom(R) = \{x \in A\ |(x, y) \in\ R\ for\ some\ y\ \in B\}}$

Range of R: the set of all the second components of the elements of R

$\boldsymbol{Ran(R) = \{y \in B\ |(x, y) \in\ R\ forsome\ \ x\ \in\ A\}}$.

Example: 2

1. In a certain city there are 5 secondary schools. the number of mathematics teachers taught in each school are listed in the table as shown below:

School Name	Number of math teachers
School 1	23
School 2	18
School 3	27
School 4	19
School 5	31

Table1.1:

a) Find the relation defined by the given table ?

b) Determine domain and range of the relation?

Solution:

a). In the table school names are the first components while number of mathematics teachers are taken to be the second components of the relation, then

R = {(school 1, 23), (school 2, 18), (school 3, 27), (school 4, 19), (school 5, 31)}

b). Domain (R) = {school 1, school 2, school 3, school 4, school 5}

Range (R) = {18, 19, 23, 27, 31}

2. Let R is a relation defined as $R = \{(x, y): y = 2x^2 - 1\}$ for $x \in \{-2, -1, -0.1, 0, 1, 2\}$ then Find domain and range of R?

Solution:

Let the value of x be any real number and y is determined by substituting x in the formula $y = 2x^2 - 1$ as follows:

When $x = -2,\ y = 2(-2)^2 - 1 = 8 - 1 = 7$

When $x = -1, y = 2(-1)^2 - 1 = 2 - 1 = 1$

When $x = -0.1, y = 2(-0.1)^2 - 1 = 0.02 - 1 = -0.98$

When $x = 0, y = 2(0)^2 - 1 = 0 - 1 = -1$

When $x = 1, y = 2(1)^2 - 1 = 1$

When $x = 2, y = 2(2)^2 - 1 = 7$

Then R = {(-2,7), (-1,1), (-0.1, -0.98), (0, -1), (1,1), (2, 7)}

Domain (R) = {-2, -1, -0.1, 0, 1, 2}

Range (R) = { -1, -0.98, 1, 7}

1.2. Representation of relations

A relation is represented by either of :

- **Set of ordered pairs**
- **Correspondence between domain and range**
- **Graph**
- **Equations**
- **An inequality or combination of any of these.**

A. Set of ordered pairs:

Example 3: Let R be a relation defined by $R = \{(1,2), (3,4), (5,6)\}$ determine domain and range of R

Solution: Domain = {1, 3, 5} is the set of first components and

Range = {2, 4, 6} is the set of the second components

B. Correspondence between domain and range:

Example 4: A and **B** are two given sets and the relation from set A to B is given by using the diagram below, determine the relation R and find domain and range ?

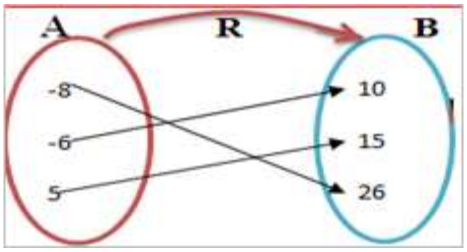

Figure 1.1

Solution: From the given diagram, the relation as a set of ordered pairs is given as:

R = {(-8,26),(-6,10), (5,15)} Whereas

Domain = {-8, -6, 5} and Range = {10, 15, 26}

C. Graph:

Example 5: Find the domain and Range of the relation given by the graph below?

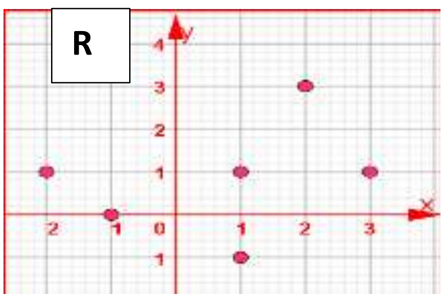

Figure 1.2

Solution :
R is represented as the set of ordered pairs of x and y.
R = {(-2, 1), (-1,0), (1,-1), (1,1), (2,3),(3,1)}
And
Domain (R) = {-2, -1, 1, 2, 3}
Range (R) = {-1, 0, 1, 3}

D. Equations

Example 6: A relation R is defined by $R = \{(x, y): x \in R, y \in R \text{ } and \text{ } y = 3x + 1\}$ find domain and range of R?

Solution:

R is defined as an infinite set of ordered pairs. The first coordinate can be any of the set of real numbers while the second coordinate becomes a real number for which it is determined by the first coordinate. As $y \in R, then\ x = \frac{y-1}{3} \in R$ or $x \in R, then\ y = 3x + 1 \in R$

Therefore Domain = the set of all real numbers

Range = the set of real numbers

E. Inequalities (region):

Example 7:

Given a. $R = \{(x, y): y \geq x^2 - 1 \text{ } and \text{ } y = 3\}$

b. $R = \{(x, y): y \leq x + 1, y \leq -x + 1 \text{ } and \text{ } y > -4\}$

i. Draw the graph of each relation

ii. Find Domain and Range

Solution:

Steps to draw graphs

- ✓ Identify x- intercepts and y- intersepts
- ✓ Identify broken and solid lines
- ✓ Connect intercepts with broken or solid lines
- ✓ Identify shadedregion

a. $\boldsymbol{R = \{(x, y): y \geq x^2 - 1 \text{ and } y \leq 3\}}$

Step1: - x and y – intecepts

❖ $\underline{\boldsymbol{x - \text{ intercept where } y = 0}}$

$y = x^2 - 1$

$0 = x^2 - 1$

$x^2 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

$\underline{\boldsymbol{y - \text{intercepts where } x = 0}}$

$y = x^2 - 1$

$y = 0 - 1$

$y = 1$

Step 2: line type, solid line and draw the graph

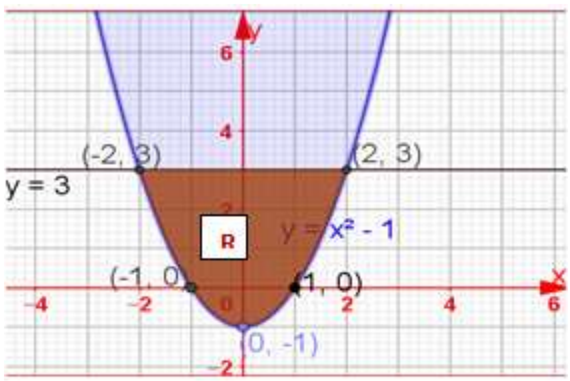

Figure 1.3

Then we just determine domain and range by:

Step 3: Find intersection points:-

$y = x^2 - 1 \text{ and } y = 3$

$\Rightarrow x^2 - 1 = 3$

$\Rightarrow x^2 = 3 + 1$

$\Rightarrow x^2 = 4 \text{ then } x = \pm\sqrt{4}$

$\Rightarrow x = \pm 2$

Domain of $R = \{x: -2 \leq x \leq 2\}$

Range of $R = \{y: -1 \leq y \leq 3\}$

b. $\boldsymbol{R = \{(x, y): y \leq x + 1, y \leq -x + 1 \text{ and } y > -4\}}$

Solution:

Intersection point:

$y = x + 1, y = 1 - x \text{ and } y = -4$ intersection point of lines $y = 1 - x \text{ and} y = -4$

intersection of lines $y = x + 1 and\ y = 1 - x$

$\Rightarrow x + 1 = 1 - x$

$\Rightarrow x + x = 1 - 1$

$.\Rightarrow 2x = 0 \ \Rightarrow x = 0\ and\ y = 0 + 1 \Rightarrow y = 1$

$.\Rightarrow intersection\ point\ (0,1)$

$.\Rightarrow -4 = 1 - x \Rightarrow x = 1 + 4 \Rightarrow x = 5$

$.intersection\ point (5, -4)$

Intersection point of lines: $y = x + 1\ and\ y = -4$

$.x + 1 = -4 \Rightarrow x = -4 - 1 \Rightarrow x = -5$

And $intersection\ point (-5, -4)$

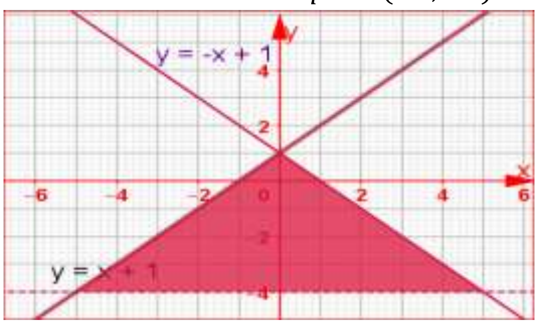

Figure 1.4

Therefore i. domain of R= $\{x: -5 \leq x \leq 5\}$ ii. Range of R= $\{y: -4 < y \leq 1\}$