Lessson 2: Inverse of Relations and Their Graph

1.1. Inverse of relation

Definintion: the inverse of relation R denoted by R^{-1} is a relation obtained by interchanging components of ordered pairs of R.

i.e. $R^{-1} = \{(y, x): (x, y) \in R\}$

Example 1:

1. Find the inverse, domainof R^{-1} and range of R^{-1} for the following relations
a. Given the relation $R = \{(-3,4), (6,12), (8,25)\}$
b. In the relation R= $\{(x, y): x \text{ is } a \text{ mother of } y\}$
c. Given $R = \{(x, y): y = 2x - 5\}$
d. R= $\{(x, y): y < 3x \text{and } y \geq 2x + 1\}$

Solution:

General: Steps to find R^{-1}

1. Interchanging coordinates is enough or
2. After interching variables solve y interms of x
3. Fix R^{-1}

a.   Interchanging components of R gives $R^{-1} = \{(4, -3), (12, 6), (25, 8)\}$
Domain of $R = \{-3, 6, 8\} \text{and range of } R = \{4, 12, 25\}$
Then domain of $R^{-1} = \{4, 12, 25\} \text{and range of } R^{-1} = \{-3, 6, 8\}$
b. R= $\{(x, y): x \text{ is } a \text{ mother of } y\}$
Domain of R = {x: x is a child } and range of R = { y : y is a mother }
And $R^{-1} = \{(y, x): x \text{ is a mother of } y\}$ or $R^{-1} = \{(x, y); y \text{ is a child of } x\}$
Domain of $R^{-1} = \{x; x \text{ is a mother}\} \text{and range of} R^{-1} = \{y: y \text{ is achild}\}$
c. $R = \{(x, y): y = 2x - 5\}$
$R^{-1} = \{(y, x): y = 2x - 5\}$
And $x = 2y - 5 \Rightarrow 2y = x + 5 \Rightarrow y = \frac{x+5}{2}$
Therefore $R^{-1} = \{(x, y): y = \frac{x+5}{2}\}$
Domain of $R^{-1} = range \text{ of } R^{-1} = \mathbb{R}$ since R^{-1} is linear

d.   R= $\{(x, y): y < x + 2 \text{ and } y \geq x + 1\}$
$R^{-1} = \{(x, y): x < y + 2 \text{ and } x \geq y + 1\}$
$= \{(x, y): y > x - 2 \text{ and } y \leq x - 1\}$
In general
Domain of $R^{-1} = Range \text{ of } R$
Range of $R^{-1} = Domain \text{ of } R$

1.2. Graph of inverse relations

Graphs of R and R^{-1} are mirror images of each other under the line y = x

Example 2:

Find the inverse of the following relations and draw graph of R and R^{-1}

a. R = {(1, 3), (2,3), (3,3)}
b. $R = \{(x,y): y = 2x - 5\}$
c. R= $\{(x,y): y < x + 2 \text{ and } y \geq x + 1\}$

Solution:

a. $R^{-1} = \{(3, 1), (3,2),(3,3)$
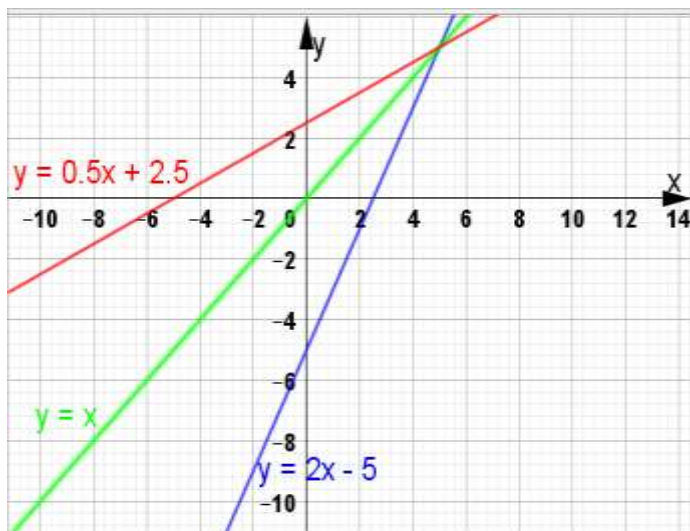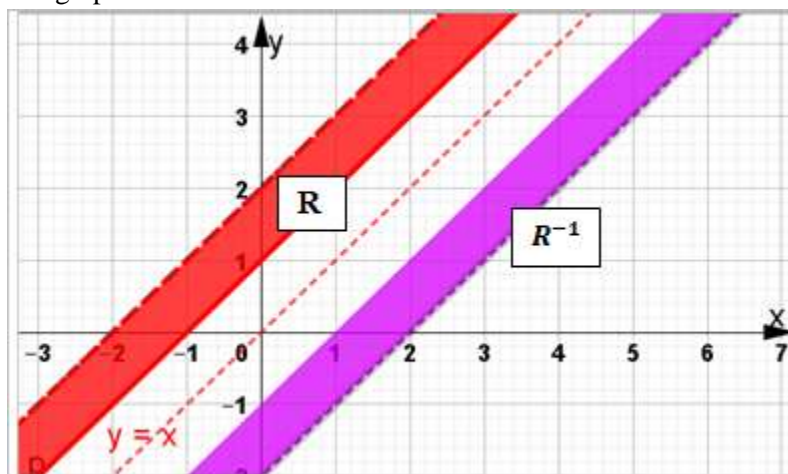

Figure 1.7

d. R= $\{(x,y): y < x + 2 \text{ and } y \geq x + 1\}$
$R^{-1} = \{(x,y): x < y + 2 \text{ and } x \geq y + 1\}$
$= \{(x,y): y > x - 2 \text{ and } y \leq x - 1\}$

Their graph looks to be:

Detrmine domain and range of R^{-1} by your own?