Lesson 3: Operations on Sets

Brainstorming Questions

Suppose Collect different flower colors in each pair and build sets from those flowers according to their color. Thus, Feyisa collects { red, Tulip, purple, yellow and white} flowers and Semira also collects { red, green, rose, yellow, white, sunflower, bluebell, and lotus} flowers from flower supermarket. Based on the above sets, build the following sets:

(a) Set of colours of flowers collected by Feyisa or Semira or both of them (Feyisa Union Semira),
(b) Set of flowers of common colours collected by Feyisa and Semira (Feyisa Intersection Semira)
(c) Set of colours of flowers collected by Feyisa only (Feyisa difference/less Semira)
(d) Set of colours of flowers except the colours of flowers collected by Semira (Semira compliment).
(e) Set of colours of flowers collected by Semira but not collected by Feyisa or Set of colours of flowers collected by Feyisa but not collected by Semira (Feyisa symmetric difference Semira)

Union of sets

A union of A and B contains all the elements in A or B or both sets. The set notation used to represent the union of sets is ∪. The set operation, set combination or union, is represented by:

$A \cup B = \{x \in A \text{ or } x \in B\}$. Here, x can be found in both sets A and B.

Example: let set A= {a, b, c, d} and B = { c, d, e, f, g}. Then, determine $A \cup B$.

Answer: $A \cup B = \{ a, b, c, d, e, f, g \}$

Intersection of sets

The intersection of two sets A and B is denoted by $A \cap B$, which is the set of all elements stated in both set A and set B.

We write this in mathematics: $A \cap B = \{x \in A \text{ and } x \in B\}$

Example: let set A= { a, b, c, d } and B = { c, d, e, f, g}. Then determine $A \cap B$.

Answer: $A \cap B = \{ c, d\}$

Complement of sets

Relative Complement (set difference) $(\setminus)$:

The difference of two sets A and B is defined as the lists of all the elements that are in set A but that are not present in set B. The set notation used to represent the difference between the two sets A and B is $A - B$ or $A \setminus B$.

Example: let set A= { a, b, c, d} and B = { c, d, e, f, g }. then determine i. $A\setminus B$ ii. $B\setminus A$.

Answer: i. $A - B = \{ a, b\}$ ii. $B - A = \{ e, f, g\}$

Absolute Compliment (')

A’ is the set which contains all the elements of the universal set other than the set itself (set A).

Example: let set A= { a, b, c, d} and B = { c, d, e, f, g} and U = { a, b, c, d, e, f, g, h} then determine i. A' ii. B'

Answer: i. $A' = \{ e, f, g, h\}$ ii. $B' = \{ a, b, h\}$

De Morgan’s Law

For the complement set of $A \cup B$ and $A \cap B$,

1st statement: $(A \cup B)' = A' \cap B'$,

2nd statement: $(A \cap B)' = A' \cup B'$.

For any two sets A and B, each of the following holds true.

i. $(A')' = A$ ii. $A' = U - A$ iii. $A - B = A \cap B'$ iV. □ □ □ □ □′ □ □′

Symmetric Difference of Two Sets

The set which contains the elements which are either in set A or in set B but not in both is called the symmetric difference between two given sets. It is represented by A Δ B and is read as a symmetric difference of set A and B.

$A \Delta B = (A - B) \cap (B - A)$

Or $A \Delta B = (A \cup B) - (A \cap B)$

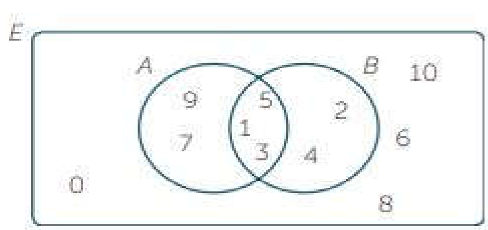
Example: let set A= { a, b, c, d } and B = { c, d, e, f, g } and U ={ a, b, c, d, e, f, g, h} then determine A Δ B?

Answer: A Δ B = { a, b, e, f, g}

Venn - Diagram

Venn diagrams are diagrams used to represent sets, diagramming the relationships between sets and operations. Introduced by John Venn (1834-1883), the Venn diagram uses circles (overlapping, intersecting, and disjoint) to show the relationships between circles.

Example 1:-



from the Venn diagram above find

i. **A∪B**: The union of sets A and B. This represents all the elements that are in either set A or set B, or in both.

ii. **A∩B**: The intersection of sets A and B. This represents all the elements that are common to both set A and set B.

iii. **A/B**: The difference of sets A and B, also written as A\ B. This represents all the elements that are in set A but not in set B.

iv. **B/A**: The difference of sets B and A, also written as B\ A. This represents all the elements that are in set B but not in set A.

v. **A△B**: The symmetric difference of sets A and B. This represents all the elements that are in either set A or set B, but not in both.

vi. **A′**: The complement of set A. This represents all the elements that are not in set A, assuming a universal set that contains all possible elements under consideration.

Answer:

i. **A∪B={1,2,3,4,5,7,9}**: The union of sets A and B is the set of all elements that are in either A, B, or both. In this case, the union includes the elements 1, 2, 3, 4, 5, 7, and 9.

ii. **A∩B={1,3,5}**: The intersection of sets A and B is the set of all elements that are common to both A and B. In this case, the intersection includes the elements 1, 3, and 5.

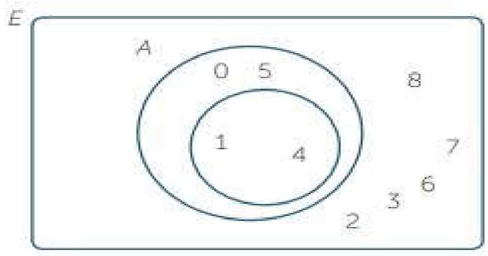
iii. **A/B={7,9}**: The difference of sets A and B (also written as A\B) is the set of all elements that are in A but not in B. In this case, the difference includes the elements 7 and 9.

iv. **B/A={2,4}**: The difference of sets B and A (also written as B\A) is the set of all elements that are in B but not in A. In this case, the difference includes the elements 2 and 4.

v. **A△B=(A\B) ∪ (B\A) = {2,4,7,9}**: The symmetric difference of sets A and B is the set of all elements that are in either A or B, but not in both. In this case, the symmetric difference includes the elements 2, 4, 7, and 9.

Example 2:- construct a Venn - diagram using the sets A={ 0, 1, 4, 5} and B ={ 1, 4} and U = {0, 1, 2, 3, 4, 5, 6, 7, 8}.

Answer:



Cartesian product of Two Sets

The Cartesian product S × T of S and T consists of all ordered pairs (s, t) such that s S and t T. Ordered pairs are characterized by the following property: (a, b) = (c, d) if and only if. a = c and b = d.

Note that: S × T is not the same as T × S except S = T.

Example 1: Given:

- Set S={a, b, c, d}
- Set T={c, d, e, f, g}
- Universal set U={a, b, c, d, e, f, g, h}

Answer:

i. S × T ={(x, 1),(x, 2),(x, 3),(y, 1),(y, 2),(y, 3)}: The Cartesian product A×B is the set of all ordered pairs where the first element comes from set A and the second element comes from set B. However, the given ordered pairs (x, 1),(x, 2),(x, 3),(y, 1),(y, 2),(y, 3) do not correspond to the elements of sets A and B. Instead, it seems like you intended to describe a different relationship, but if this was the example you meant, you might want to correct the elements involved.

ii. **T × S ={(1, x),(2, x),(3, x),(1, y),(2, y),(3, y)}**: Similarly, the Cartesian product B×A is the set of all ordered pairs where the first element comes from set B and the second element comes from set A. The pairs (1,x),(2,x),(3,x),(1,y),(2,y),(3, y) also do not match the elements of sets B and A.

Example 2: Given:

- Set A={a, b, c, d}
- Set B={c, d, e, f, g}

- Universal set U={a, b, c, d, e, f, g, h}

Answer:

- **A×B** would actually be the set of all ordered pairs where the first element comes from A and the second from B, which would look like:

A×B={(a, c), (a, d), (a, e), (a, f), (a, g), (b, c), (b, d), (b, e), (b, f), (b, g), (c, c), (c, d), (c, e), (c, f), (c, g), (d, c), (d, d), (d, e), (d, f), (d, g)}.

- **B×A** would be the set of all ordered pairs where the first element comes from B and the second from A, which would look like:

B×A={(c, a), (c, b), (c, c), (c, d), (d, a), (d, b), (d, c), (d, d), (e, a), (e, b), (e, c), (e, d), (f, a), (f, b), (f, c), (f, d), (g, a), (g, b), (g, c), (g, d)}.

These describe the relationships between all possible pairs formed by combining elements of the two sets.

Example 3: Find the value of x and y if (x + 1, 7) = (4, y + 2)?

Solution: To find the values of x and y from the equation (x+1,7)=(4,y+2), we compare the corresponding elements of the ordered pairs.

We have:

$x + 1 = 4$ and $7 = y + 2$

Solving for x:

$x + 1 = 4$

Subtract 1 from both sides:

$x = 4 - 1$ $\quad\quad\quad\quad$ $x = 3$

Solving for y:

$7 = y + 2$

Subtract 2 from both sides:

$y=7-2$ $y=5$

Final Answer:

The values are $x = 3$ and $y = 5$.