Lesson 3: Irrational Numbers

Specific objectives

Classify various decimals as either terminating, repeating, or irrational by analyzing their patterns

Distinguish between rational and irrational numbers by analyzing given examples, including square roots and decimals, to determine if they can be expressed as fractions or not.

Apply operations involving irrational numbers, such as addition, subtraction, multiplication, and division, and **simplify** the results to their most reduced forms.

Locate and **plot** irrational numbers such as $\sqrt{2}$ and $\sqrt{3}$ on a number line by estimating their approximate positions relative to known rational numbers.

Brainstorming questions

Here are three real-life brainstorming questions related to the concepts of rational and irrational numbers, tailored to Ethiopian daily activities:

1. **Question**: "When buying fruits from a market in Ethiopia, you notice that some fruits are sold by weight, and the prices are given as decimals. How can you determine if a price expressed in decimal form is a rational number? For example, if apples cost 0.75 birr per gram, is this price a rational number? Why or why not?"

Explanation:
To determine if a price expressed as a decimal is a rational number, check if it can be expressed as a fraction. In this case, 0.75 can be written as $\frac{75}{100}$, which simplifies to $\frac{3}{4}$. Since it can be expressed as a fraction of two integers, it is a rational number. Prices in decimal form at markets are typically rational because they can be converted into fractions representing their exact values.

2. Question: "When constructing traditional Ethiopian houses, measurements are often made using irrational numbers. For instance, if you need to measure the diagonal of a square room with

each side of 5 meters, the length of the diagonal is $\sqrt{50}$ meters. How would you simplify $\sqrt{50}$ to use in practical construction, and why is $\sqrt{50}$ an irrational number?"

Explanation:

To simplify $\sqrt{50}$, factor it into $\sqrt{25 * 2}$ which equals $(5\sqrt{2})$. This simplification helps in practical applications by giving a more manageable number. $\sqrt{50}$ is irrational because it cannot be expressed as a simple fraction of two integers. The decimal representation of $(\sqrt{50})$ is non-repeating and non-terminating, thus making it an irrational number.

3. Question: "In Ethiopian cooking, recipes might require precise measurements. If a recipe calls for 0.2 liters of a liquid ingredient, is this quantity a rational number? What about if you need to measure (π) liters for a specific recipe? How would you handle these measurements in practice?"

Explanation:

0.2 liters can be expressed as $(\frac{2}{10})$ or $\frac{1}{5}$, making it a rational number because it can be written as a fraction. On the other hand, (π) liters represents an irrational number because $\pi)$ cannot be expressed as a fraction and its decimal representation is non-repeating and non-terminating. In practice, for irrational numbers like (π), you would use an approximation (e.g., 3.14) for measurements or adjust the recipe according to practical needs.

Lesson 3: Irrational Numbers

Classified as rational numbers because they can be expressed as fractions of numbers.

Definition: Irrational numbers are

- Neither repeating nor terminating numbers. Examples: 0.121221222122...., 3.125436745897...., -12.020220222022...., ...
- $\sqrt[n]{x}$, where x is not a perfect n^{th} exponent number. Example: $\sqrt{10}$, $\sqrt[3]{16}$, $\sqrt[4]{8}$, ...
- π as itself is irrational number. But, the rational approximation numbers 3.14 and $\frac{22}{7}$ are rational numbers.

Example: Determine whether each of the following numbers is rational or irrational.

a) $\sqrt{36}$ b) $\sqrt{46}$ c) $\sqrt{0.09}$ d) $\sqrt{0.16}$

Answer: a) $\sqrt{36} = \sqrt{6 * 6} = 6$. Thus, $\sqrt{36}$ is rational number.

b) $\sqrt{46} = \sqrt{23 * 2}$ it can not be written out of radical. Thus, $\sqrt{46}$ is irrational number.

c) $\sqrt{0.09} = \sqrt{0.3 * 0.3} = 0.3$. thus $\sqrt{0.09}$ is rational number.

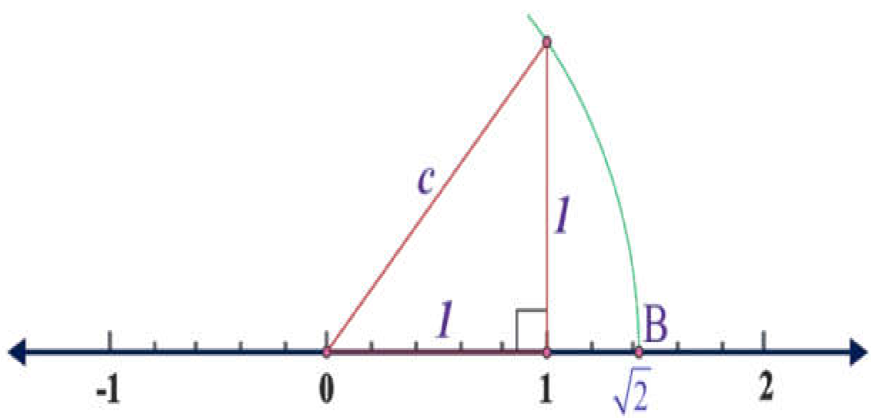
d) $\sqrt{0.16} = \sqrt{0.4 * 0.4} = 0.4$, thus $\sqrt{0.16}$ is rational number.

Locating irrational number on the number line

Example: Locate $\sqrt{2}$ on the number line.

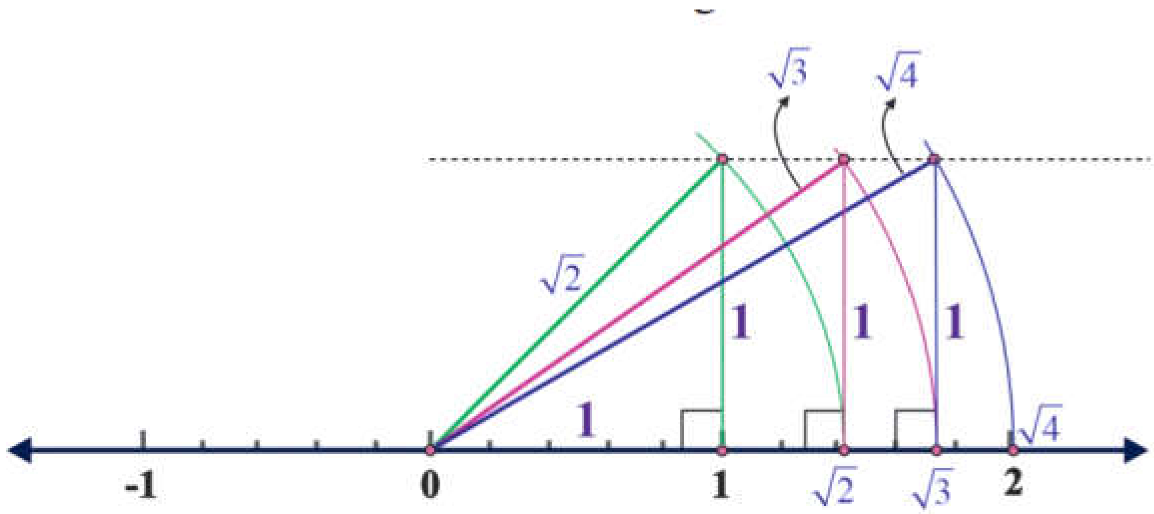
Answer:

- Draw a number line. Label an initial point 0 and points 1 unit long to the right and left of 0. Construct a perpendicular line segment 1 unit long at 1.



Example: Locate 1, $\sqrt{2}$, $\sqrt{3}$, and 2, on the number line.

Answer:



2.3.2 Operations on irrational numbers

The paragraph is informing the reader that in section 2.3.1 of the material, they have learned about what an irrational number is and how it is represented on the number line. The paragraph then states that in the upcoming section, the focus will be on discussing the operations of addition, subtraction, multiplication, and division involving irrational numbers.

When a > 0, and b > 0, then $\sqrt{a} * \sqrt{b} = \sqrt{ab}$.

Example: Find the solution for each of the following equations/problems.

a. $\sqrt{2} * \sqrt{7}$ b. $5\sqrt{3} * \sqrt{5}$ c. $\frac{3}{\sqrt{7}} * \frac{11}{\sqrt{3}}$ d. $(\sqrt{5} + \sqrt{2})(\sqrt{6} - \sqrt{5})$

Answer: a) $\sqrt{2} * \sqrt{7} = \sqrt{2 * 7} = \sqrt{14}$

b. $5\sqrt{3} * \sqrt{5} = 5\sqrt{3 * 5} = 5\sqrt{15}$

c. $\frac{3}{\sqrt{7}} * \frac{11}{\sqrt{3}} = \frac{3*11}{\sqrt{7*3}} = \frac{33}{\sqrt{21}}$

d. $(\sqrt{5} + \sqrt{2})(\sqrt{6} - \sqrt{5}) = (\sqrt{5}\sqrt{6} - \sqrt{5}\sqrt{5} + \sqrt{2}\sqrt{6} - \sqrt{2}\sqrt{5}) = -5 + \sqrt{30} + \sqrt{12} - \sqrt{10})$

Note: When a > 0, and b > 0, then $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Examples: Calculate a) $-\frac{\sqrt{8}}{\sqrt{2}}$ b) $\sqrt{\frac{9}{4}}$

Answer: a) $-\frac{\sqrt{8}}{\sqrt{2}} = -\sqrt{\frac{8}{2}} = \sqrt{4} = \sqrt{2*2} = 2$

b) $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{\sqrt{3*3}}{\sqrt{2*2}} = \frac{3}{2}$

Note: When a > 0, and b > 0, then $a\sqrt{b} = \sqrt{a^2 b}$ and $\sqrt{a^2 b} = a\sqrt{b}$

Example: Convert each of the following in $\sqrt{d}$ form. a) $3\sqrt{5}$ b) $-4\sqrt{7}$

Answer: a) $3\sqrt{5} = \sqrt{3^2 * 5} = \sqrt{9 * 5} = \sqrt{45}$

b) $-4\sqrt{7} = -\sqrt{4^2 * 7} = -\sqrt{16 * 7} = -\sqrt{112}$.

Example:- Simplify each of the following.

a. 0.454554555 … + 0.32322322 … c) $\sqrt{18} + \sqrt{50}$

b. 0.363663666 … − 0.141441444 … d) $\sqrt{48} - \sqrt{27}$

Answer: a) 0.454554555 … + 0.32322322 … = 0.777777........

b) 0.363663666 … − 0.141441444 … = 0.22222......

c) $\sqrt{18} + \sqrt{50} = \sqrt{3*3*2} + \sqrt{5*5*2} = 3\sqrt{2} + 5\sqrt{2} = (3+5)\sqrt{2} = 8\sqrt{2}$

d) $\sqrt{48} - \sqrt{27} = \sqrt{2*2*2*2*3} - \sqrt{3*3*3} = 2*2*\sqrt{3} - 3\sqrt{3} = 4\sqrt{3} - 3\sqrt{3} = 4 - 3)\sqrt{3} = 1\sqrt{3} = \sqrt{3}$