Lesson 4: Application

Set is a collection of objects, termed elements. The objects of the sets can be anything from people, districts, countries, or any other possibilities. Application of sets is used in statistics, Boolean algebra, and probability.

Brainstorming questions

1. In a study of 250 people, it was found that 150 people consume injera daily, 120 people consume bread daily, and 80 consume both injera and bread daily. How many people consume either injera or bread but not both?
2. In a survey of 200 people in Addis Ababa, 120 prefer using buses for transportation, 100 prefer taxis, and 60 use both buses and taxis. How many people prefer either buses or taxis but not both?

Solution

Given:- Total number of people surveyed: n = 250 Number of people who consume injera daily: n (A) = 150 Number of people who consume both injera and bread daily: n(A ∩ B) = 80 We need to find the number of people who consume either injera or bread but not both.

Step 1: Calculate the total number of people who consume either injera or bread (or both).

n (A ∪ B) = n(A) + n(B) – n(A ∩ B) = 150 + 120 - 80 = 190

So, 190 people consume either injera or bread (or both).

Step 2: Calculate the number of people who consume either injera or bread but not both. The number of people who consume only injera: n(A) – n(A ∩ B) = 150 - 80 = 70

The number of people who consume only bread: n(B) – (A ∩ B) = 120 - 80 = 40

Step 3: Add the people who consume only injera and those who consume only bread. {People who consume either injera or bread but not both} = 70 + 40 = 110

Final Answer: The number of people who consume either injera or bread but not both is 110. 2. Given: Total number of people surveyed: (n = 200)

Number of people who prefer buses: (n(A) = 120)

Number of people who prefer taxis: (n(B) = 100)

Number of people who prefer both buses and taxis: (n(A ∩ B) = 60)

We need to find the number of people who prefer either buses or taxis but not both.

Step 1: Find the number of people who prefer either buses or taxis (or both). n(A ∪ B) = n(A) + n(B) – n(A ∩ B) = 120 + 100 - 60 = 160

So, 160 people prefer either buses or taxis (or both).

Step 2: Find the number of people who prefer either buses or taxis but not both. The number of people who prefer buses only: n(A) – n(A ∩ B) = 120 - 60 = 60

The number of people who prefer taxis only: n(B) – n(A ∩ B) = 100 - 60 = 40

Step 3: Add the people who prefer buses only and those who prefer taxis only. {People who prefer either buses or taxis but not both} = 60 + 40 = 100

Final Answer: The number of people who prefer either buses or taxis but not both is 100.

Note: For any two finite sets A and B,

i. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$,

ii. n(A/B) = n(A) – n (A n B).

iii. $n(A \Delta B)$ = n(A) + n(B) - 2n(A n B)

Examples

1. In a class of 31 students, 22 students study physics, 20 students study chemistry and 5 students study neither. Calculate the number of students who study both subjects

2. Suppose A and B are sets such that A ⋃ B has 20 elements, A ⋂ B has 7 elements, and the number of elements in B is twice that of A. What is the number of elements in: a) A? b) B?.

3. Of 100 staff members of a school, 48 drink coffee, 25 drink both tea and coffee and everyone drinks either coffee or tea. How many staff members drink tea?

Answers

1. Given that n(U) = 31, n(P) = 22, n(C) = 20, n(P U C)' = U - (P U C) = 5 Required: n(P n C)= ?

Answers: n(P U C) = U - n(P U C)' = 31 - 5 = 26, n(P U C) = n(P) + n(C) - n(P n C), 26 = 22 + 20 - n(P n C), Thus, n(P n C) = 16,

Therefore there are 16 students who study both physics and chemistry.

2. Given that n (A U B) = 20, n (A n B) = 7, n (B) = 2 n(A) Required: a) A? b) B?

Answers: n (A U B) = n(A) + n (B) - n (A n B), n (A U B) = n(A) + 2 n(A) - n (A n B), n (A U B) = 3n(A) - n (A n B), 20 = 3n(A) - 7,

3n(A) = 27, n (A) = 9, on the other hand n (B) = 2 n(A) = 2 (9) = 18, thus n(B) = 18

3. Given that n(U) = 100, n(C) = 48, n(C n T) = 25, n(P U C) = U = 100, Required: n(T) = ?

Answers: n(C U T) = n(C) + n(T) - n(C n T), 100 = 48 + n(T) - 25, n(T) = 100 - 48 + 25 = 77,

Therefore 77 staff members who drink tea.