Lesson 2: The Notion of Sets

Empty set, Finite set, Infinite set, Power set, Subset and Proper subset

1. Empty Set: An empty set is a set that contains no elements. It is denoted by { } or Ø.
Example, a) The set of whole numbers between 2 and 3. b) The set of negative natural numbers.

2. Finite Set: A finite set is a set that has a specific number of elements.
Example, a) The set of colors in a rainbow {red, orange, yellow, green, blue, indigo, violet} is a finite set because it has a limited number of elements. b) The set of peoples in the world. c) The set of trees in Africa. d) The natural number less than 1000.

3. Infinite Set: An infinite set is a set that has an unlimited number of elements.
Example, a) The set of natural numbers {1, 2, 3, ...} is an infinite set because it continues indefinitely. b) The set of real numbers between 0 and 1.

4. Power Set: The power set of a set is the set of all possible subsets of that set, including the set itself and the empty set.
Example, The power set of the set { a, b } is {{}, {a}, {b}, {a, b}}.
"In mathematics, the number of elements in a set A is represented by (A). This notation (A) simply tells us how many distinct elements are contained within the set A. For **Example**, if set A consists of the numbers {1, a, 3, b}, then (A) would be 4 because there are four elements in the set.

"In set A = {2, 3, 4}, there are 3 elements: 2, 3, and 4. this means that the number of elements of set A is 3.

The power set of a set is the set of all possible subsets that can be formed from the original set. For set A = {2, 3, 4}, the power set includes the following subsets: {}, {2}, {3}, {4}, {2, 3}, {2, 4}, {3, 4}, and {2, 3, 4}.

The number of elements in the power set of set A can be calculated using the formula 2^n, where n is the number of elements in the original set. In this case, n = 3, so 2^3 = 8. Therefore, the power set of set A contains 8 elements, representing all possible combinations of elements from the original set."

Example: The power set of the set {a, b, c, 4} is {{}, {a}, {b}, {c}, {4}, {a, b}, {a, c}, {a, 4}, {b, c}, {b, 4}, {c, 4}, {a, b, c}, {a, b, 4}, {a, c, 4}, {b, c, 4}, {a, b, c, 4}}.

5. Subset: set A is a subset of set B if every element of A is also an element of B. This is denoted as A ⊆ B.

Note

1) Any set is a subset of itself.
2) The empty set is a subset of every set.
3) If set A is finite with n elements, then the number of subsets of set A is 2^n.

Example, a) the set {red, green} is a subset of the set {red, green, blue} because all its elements are also in the larger set.

Example b) Set A = {1, 2, 3} and Set B = {1, 2, 3, 4, 5}. Ask students to identify if Set A is a subset of Set B. (Answer: A ⊆ B)

Example c) Set C = {red, blue, green} and Set D = {red, blue, green, yellow}. Ask students to determine if Set C is a subset of Set D? yes set C is a subset of set D.

For any set A with n number of elements. Then, set A has a total of 2^n number of subsets.

Example: set A = {1, 2, 3}, set A has three elements (n = 3). thus, set A has a total of 2^n number of subsets. Set A has 3 elements, 2^3 = 8 sets. these eight subsets of set A are {}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}. Which means {} ⊆ {1, 2, 3}, {1} ⊆ {1, 2, 3}, {2} ⊆ {1, 2, 3}, ..., {1,2,3} ⊆ {1, 2, 3}

6. Proper Subset: A proper subset is a set that contains some, but not all, of the elements of another set. In other words, if set A is a proper subset of set B, then all elements of set A are also elements of set B, but set B contains at least one element that is not in set

A. This distinction is important because if two sets are equal, they are not considered proper subsets of each other.

Note

1) For any set A, A is not a proper subset of itself.
2) The number of proper subsets of set A is 2^n - 1, where n is the number of elements in set A.
3) The empty set is a proper subset of any other set.
4) If set A is a subset of set B (A ⊆ B), then conversely, B is the superset of A, denoted as B ⊃ A.

Example, the set {red, green} is a proper subset of the set {red, green, blue} because it does not include all the elements of the larger set.

For any set A with n number of elements. Then, set A has a total of 2^n - 1 number of proper subsets.

Example: set A = {1, 2, 3}, set A has three elements (n = 3). thus, set A has a total of 2^n - 1 number of subsets. Set A has 3 elements, 2^3 - 1= 8 - 1 = 7 sets. these seven proper subsets to set A are {}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}. Which means {} □ {1, 2, 3}, {1} □ {1, 2, 3}, {2} □ {1, 2, 3}, ..., {2,3} □ {1, 2, 3}.

Equal Sets: Equal sets refer to sets that have exactly the same elements. In other words, two sets are considered equal if they contain the same elements, regardless of the order in which they are listed. Mathematically, it is denoted as A = B.

Example, the sets A = {1, 2, 3} and B = {3, 2, 1} are equal sets because they contain the same elements (A = B).

Equivalent Sets: Equivalent sets refer to sets that have the same number of elements. While the elements themselves may differ. On the other hand if two sets are equivalent, then n(A) = n(B). This is written mathematically as A ↔ B (or A~B). if set A is a subset of set B (denoted as A ⊆ B) and A is not equal to B, then A is considered a proper subset of B. This relationship can be represented as A ⊂ B.

Example: the sets A = {1, 2, 3} and B = {a, 5, y} are equivalent sets because they both contain three elements (A ↔ B or A ~ B).

Universal Set: The universal set, also known as the sample space, is a set that contains all the elements under consideration in a particular context. It is denoted by the symbol U. The universal set helps define the boundaries within which other sets are defined and compared.

Example, if we are considering a set of all students in a school, the universal set would include every student enrolled in that school.