Lesson 1: Revision on Natural Numbers and Integers

Brainstorming questions

1. How many birr will three workers receive as a quotient from 700 birr, and how many birr will be left as a remainder?
2. For how many workers can 1000 birr be divided equally without any cents? A)2 B)3 C)4 D)5 E)6 F)7 G) 8 H)9 I) 10
3. for how many workers does 97 birr be divided? if birr 97 is divided by only 1 worker or 97 worker, then 97 is prim number? check!
4. Find the Greatest Common Factor (GCF) of 36 and 30, and then simplify 36/30.
5. If Gebremeskel takes 40 minutes to cover one full truck and Kenenisa takes 12 minutes to cover one full track, how many rounds will they complete before meeting at the starting point?

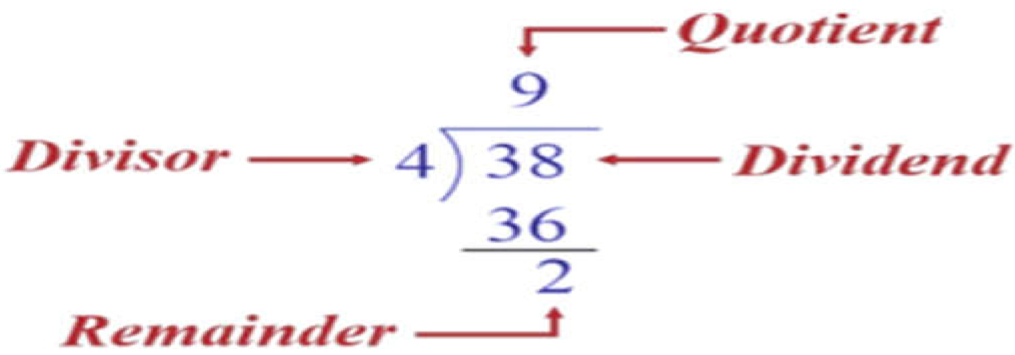
- **From your grade 7 mathematics lessons, you recall that**
- The set of natural numbers, denoted by $\mathbb{N}$, consists of all positive integers starting from 1, it can be expressed as $\mathbb{N} = \{1, 2, 3, ...\}$.
- Whole numbers are a set of non-negative integers. In mathematical terms, the set of whole numbers is denoted by the symbol W and can be expressed as W = {0, 1, 2, 3, ...}.
- The set of integers, denoted by $\mathbb{Z}$. In set notation, integers can be expressed as $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, 3, ...\}$.

2.1.1 Euclid's Division lemma

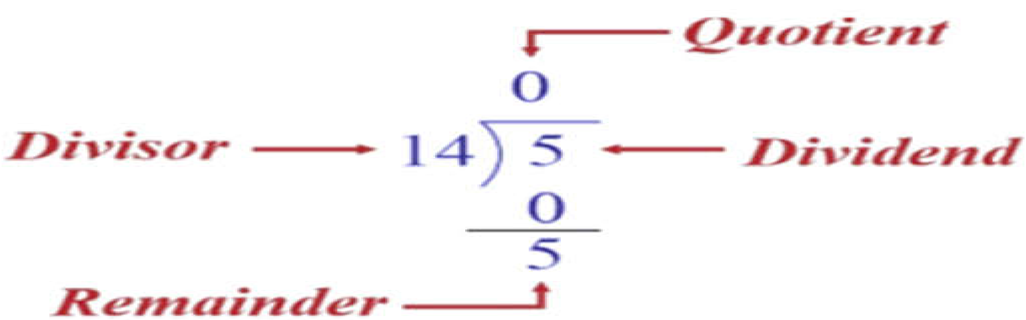
Definition: Euclid's Division Lemma states that for any two positive integers a and b, there exist unique integers q and r such that a = bq + r, where 0 ≤ r < b.

where a is called the dividend, q is called the quotient, b is called the divisor, and r is called the remainder.

Example: Find the unique quotient and remainder when a positive integer. a) 38 is divided by 4 b) 5 is divided by 14



Solution: Thus, the quotient is 9 and the remainder is 2.



Solution: Thus, the quotient is 0 and the remainder is 5.

In the Division Lemma, for two positive integers a and b, we say that a is divisible by b if the remainder r is zero.

2.1.2 Prime numbers and composite numbers

Prime numbers are positive integers greater than 1 that have exactly two distinct positive divisors: 1 and the number itself.

Example: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ... because of those numbers are exactly divisible without remainder by the number 1 and the number itself.

C**omposite numbers** are positive integers greater than 1 that have more than two distinct positive divisors.

Example: 4, 6, 8, 9, 10, 12, 14, 15, 16, ... because of those numbers are exactly divisible without remainder by more than two distinct factors. let, take 9. 9 is divisible by 1, itself (9), and 3. there are 3 factors exactly divided the number 9. Thus, 9 is composite number.

- **Note**
- 1 is neither prime nor composite
- 2 is the only even prime number, as it is divisible only by 1 and itself.
- Factors of a number are always less than or equal to the number itself.
- If the difference between two prime numbers is two. then, the two numbers is twin prime numbers.
- Examples of twin prime numbers are 3 and 5, 5 and 7, 11 and 13, 17 and 19, 29 and 31, ... those pairs of numbers are twin primes.

- Given two natural numbers h and p, h is considered a multiple of p if there exists a natural number q such that $h = p \times q$. In this context:
    - p is referred to as a factor or divisor of h.
    - h is said to be divisible by p.
    - q is also a factor or divisor of h.
    - h is divisible by q.
- Therefore, for any two natural numbers h and p, h is divisible by p if there is a natural number q such that $h = p \times q$.
- Example:

2.1.3 Divisibility test

A number is divisible by:

- A number is divisible by 2, if its unit digit is divisible by 2.
- A number is divisible by 3, if the sum of its digits is divisible by 3.
- A number is divisible by 4, if the number formed by its last two digits is divisible by 4.

- 5, if its unit digit is either 0 or 5.
- 6, if it is divisible by 2 and 3.
- 7, the last digit of the given number should be multiplied by 2 and then subtracted with the rest of the number leaving the last digit. If the difference is 0 or a multiple of 7, then it is divisible by
- 8, if the number formed by its last three digits is divisible by 8.
- 9, if the sum of its digits is divisible by 9.
- 10, if its unit digit is 0.
- 11 if the difference between the sum of its digits in odd places and the sum of the digits in even places is either 0 or a multiple of 11.

Example : - Is the number 9,812,624 divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.

Answer: 9,812,624 is divisible by 2. because the unit digit is 4, which is divisible by 2.

9,812,624 is not divisible by 3. because the sum of the digit 9 + 8 + 1 + 2 + 6 + 2 + 4 = 32, thus, 32 is not divisible by 3.

9,812,624 divisible by 4. because the last two digits is 24, which is divisible by 4.

9,812,624 is not divisible by 5. because the last digit is not either 0 or 5, thus 9,812,624 is not divisible by 5.

9,812,624 is not divisible by 6. because the number is not divisible by both 2 and 3, specifically it is not divisible by 3. Thus not divisible by 6.

9,812,624 is not divisible by 7. because 981,262 - 2(4) = 981,254, 98,125 - 8 = 98,117, 9,811 - 14 = 9,797, 979 - 14 = 965, 96 - 10 = 86, 8 - 12 = - 4, Thus - 4 is not divisible by 7. therefore, 9,812,624 is not divisible by 7.

9,812,624 is divisible by 8. because the last three digits 624 is divisible by 8. Thus 9,812,624 is divisible by 8.

9,812,624 is not divisible by 9. because the sum of the digit 9 + 8 + 1 + 2 + 6 + 2 + 4 = 32, thus, 32 is not divisible by 9.

9,812,624 is not divisible by 10. because the unit digit is not 0, thus, 9,812,624 is not divisible by 10.

9,812,624 is not divisible by 11. because the difference between (the sum of odd digit digits) and (the sum of even digits) is not 0, look there difference (9 + 1 + 6 + 4) - (8 + 2 + 2) = 8. thus, 9,812,624 is not divisible by 11.

Prime factorization is the process of expressing a composite number as a product of prime numbers.

Example: Express number 72 as prime factorization form.

$Answer: 72 \div 2 = 36$

$36 \div 2 = 18$

$18 \div 2 = 9$

$9 \div 3 = 3$

The last number 3 is the prime number. Thus, $72 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^2$

2.1.4 Greatest common factor and least common multiple

A) Common factor and Greatest common factor

Definition: The Greatest Common Factor (GCF) of two or more numbers is the largest number that divides each of the numbers evenly. It is also known as the Greatest Common Divisor (GCD).

Given two or more natural numbers, a number that is a factor of these natural numbers is called a common factor.

If the greatest common factor of two or more numbers is 1, then the numbers are relatively prime numbers.

Example: Find the Greatest common factor (GCF) of 24 and 56 using different techniques.

Answer:

Case 1: by finding common factors and finally the greatest number from common factor is GCF.

Factor of 24 = { 1, 2, 3, 4, 6, 8, 12, 24} and factor of 56 = {1, 2, 4, 7, 8, 14, 28, 56}. Then, Common factor of 24 and 56 = {1, 2, 4, 8}.

Therefore, GCF (24, 56) = 8.

Case 2: using prime factorization method (identify the same base with minimum exponent):

$$24 = 2^3 \times 3,\ \ 56 = 2^3 \times 7,\ \ the\ GCF\ (24, 56) = 2^3 = 8$$

Case 3: using table (identify the common factors and multiply them)

Common divisors	24	56
2	12	28
2	6	14
2	3	7
No other common factor		

Thus, the GCF of 24 and 56 is $= 2 \times 2 \times 2 = 8$

Case 4: In the Euclidean algorithm, if positive integers a, b, q, and r satisfy the equation a = q × b + r, then the Greatest Common Factor (GCF) of a and b is equal to the GCF of b and r.

$$\frac{56}{24} = (24 \times 2) + 8$$

Thus, GCF (56, 24) = GCF (24, 8)

$\frac{24}{8} = 8 \times 3$ Here the remainder is zero. When the remainder is zero the GCF of the two numbers is the smallest number.

Thus, GCF (24, 8) = 8. Therefore: GCF (56, 24) = 8.

Example 3: Find the Greatest common factor (GCF) of 24, 48 and 72 using different techniques.

Case 1: Identify common factors and finally the greatest number from common factor is GCF.

Factor of 24 = {1, 2, 3, 4, 6, 8, 12, 24},

Factor of 48 = {1, 2, 3, 4, 6, 8, 12, 16, 24, 48} and

72 = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}.

Then, Common factor of 24, 48 and 72 = {1, 2, 3, 4, 6, 8, 12, 24}.

Therefore, GCF (24, 48, 72) = 24.

Case 2: using prime factorization method (identify the same base with minimum exponent):

$$24 = 2^3 \times 3$$

$$48 = 2^4 \times 3$$

$$72 = 2^3 \times 3^2$$

$$GCF\ (24, 48, 72) = 2^3 \times 3 = 24$$

Case 3: Using table (identify the common factor and multiply them)

Common Divisor	24	48	72
2	12	24	36
2	6	12	18

2	3	6	9
3	1	2	3
There is no any other common factor.			

Thus, the GCF of 24, 48 and 72 is = $2 \times 2 \times 2 \times 3 = 24$

Example 3: Find the GCF (36, 56) using Venn - diagram.

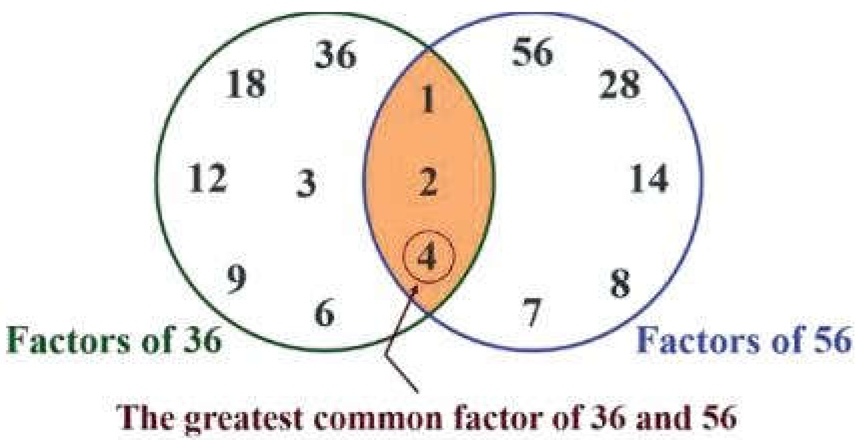


Figure 2.1 Factors of 36 and 56

Figure 2.1 is copied from fig 2.1

As you have seen in the Venn - diagram the common factors of 36 and 56 is {1, 2, 4}

Thus, the greatest from the common factors from the common factor {1, 2, 4} is 4.

Therefore, GCF (36, 56) = 4

B) Common multiple and least common multiple

Definition: Common multiples are multiples that are shared by two or more numbers. The Least Common Multiple (LCM) is the smallest multiple that is divisible by two or more numbers. It is the smallest number that is a multiple of all the numbers in question.

Given two or more natural numbers, a number that is a multiple of these natural numbers is called a common factor.

Example 1: Find the least common multiple (LCM) of 8 and 12 using different techniques.

Answer:

Case 1: by finding common multiple and finally the smallest number from common multiple is LCM.

Factor of 8 = {8, 16, 24, 32, 40, 48, 56, 64, 72, 80 .... } and

Factor of 12 = { 12, 24, 36, 48, 60, 72, 84, ...}.

Then, Common multiple of 8 and 12 = {24, 48, 72, ...}.

Therefore LCM of 8 and 12 = 24.

Case 2: using prime factorization method (identify the same base with maximum exponent and any different numbers are multiplied):

$8 = 2^3$, $12 = 2^2 \times 3$, thus, $LCM\ (8, 12) = 2^3 \times 3 = 24$

case 3: using table (identify the common factors and multiply them)

Common factors	8	12
2	4	6
2	2	3
2	1	3
3	1	1
No more factors		

Thus, the LCM of 8 and 12 is = $2 \times 2 \times 2 \times 3$ = 24

Example 3: Find the least common multiple (LCM) of 12, 8 and 16 using different techniques.

Answer: Identify common multiples and finally the least/smallest number from common multiples is Least Common Multiple (LCM).

Multiple of 12 = { 12, 24, 36, 48, 60, 72, 84, 96, …. },

Factor of 8 = {8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96,…..} and

16 = {16, 32, 48, 64, 88, 96, 112, …. }.

Then, Common multiple of 12, 8 and 16 = {48, 96, 144, …..}.

Therefore, LCM of 12, 8 and 16 = 48.

Case 2: Using prime factorization methods (identify the same base with maximum exponent and any other number finally multiply selected numbers):

$12 = 2^2 \times 3$, $8 = 2^3$ and $16 = 2^4$. Then, $LCM\ (12, 8, 16) = 2^4 \times 3 = 48$

case 3: using table (identify the common factors and multiply them)

Common Factors	12	8	16
2	6	4	8
2	3	2	4
2	3	1	2
2	3	1	1
3	1	1	1

No more factors

Thus, the LCM of 12, 8 and 16 is $= 2 \times 2 \times 2 \times 2 \times 3 = 48$

Note: For any natural numbers a and b, GCF (a, b) × LCM (a, b) = a × b

Example:- The GCF of two numbers is 6 and the LCM of these two numbers is 72. If one of the numbers is 24, what is the other number?

Solution: GCF (a, b) × LCM (a, b) = a × b

$$6 \times 72 = 24 \times b$$

$$432 = 24b$$

$$b = 18$$