UNIT 2

Lesson 2: Rational Numbers

Brainstorming Question

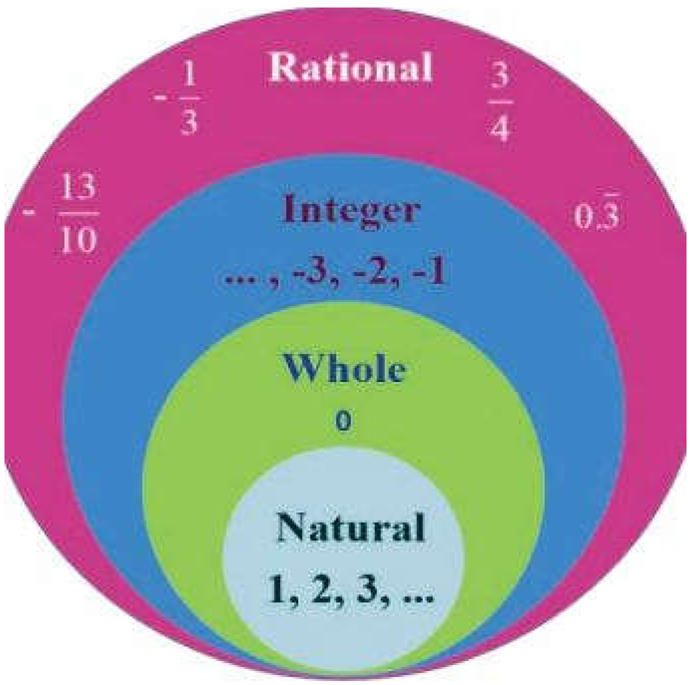

Figure 2.2

Fig 2.2 is from grade 9th students' text book.

Definition:

- Rational numbers are numbers denoted by Q.
- Rational numbers are numbers that can be expressed as a ratio of two integers, where the denominator is not zero.
- They can be written in the form of $Q = \{\frac{a}{b}\ where\ a, b\ \in Z\ and\ b\ \neq 0\}$.
- Rational numbers include natural numbers, whole numbers, integers, fractions, and terminating or repeating decimals.

Note

Suppose $x = \{\frac{a}{b} \in \mathbb{Q}\}$, x is a fraction with numerator a and denominator b. Then,
i) If $a < b$, then x is called proper fraction.
ii) If $a \geq b$, then x is called improper fraction.
iii) If y = $\frac{a}{b}$, where $c \in \mathbb{Z}$ and $\frac{a}{b}$ is a proper fraction, then y is called a mixed fraction (mixed

number).
iv) x is said to be in simplest (lowest) form if a and b are relatively prime or GCF (a, b) = 1.

Example: Categorize each of the following as proper, improper or mixed fraction $\frac{2}{3}, \frac{7}{6}$, $12, and\ 5\frac{2}{3}$.

Answer: $\frac{2}{3}$ is proper fraction, $\frac{7}{6}$ is improper fraction, 12 (or $\frac{12}{1}$) is improper fraction, and $5\frac{2}{3}$ is mixed fraction.

Example 2: Express the improper fraction $\frac{15}{7}$ as mixed fraction? Answer: $\frac{15}{7} = \frac{14+1}{7} = \frac{14}{7} + \frac{1}{7} = 2 + \frac{1}{7} = 2\frac{1}{7}$.

Example 3: Express the mixed fraction $5\frac{4}{7}$ as improper fraction? Answer: $5\frac{4}{7} = \frac{5}{1} + \frac{4}{7}$ to make denominator the same multiply $\frac{5}{1}$ by $\frac{7}{7}$ we will get $\frac{35}{7}$. Thus, add $\frac{35}{7}$ and $\frac{4}{7}$we will get $\frac{39}{7}$.

2.2.1 Representation of rational numbers by decimals

Example 1: Perform each of the following divisions. a) $\frac{4}{5}$ b) $\frac{15}{4}$ c) $-\frac{8}{3}$

Answer: The expressions you provided are fractions and can be interpreted as follows:

a) $\frac{4}{5}$: This represents the fraction four-fifths or 4 divided by 5. The result in decimal form is 0.8.

b) $\frac{15}{4}$: This is the fraction fifteen-fourths or 15 divided by 4. The result in decimal form is 3.75.

c) $-\frac{8}{3}$: This represents the negative fraction negative eight-thirds or $-$ 8 divided by 3. The result in decimal form is approximately $-$ 2.666..., where 6 repeats indefinitely (this can be written as $-2.\overline{6}$).

example 2: Write the numbers 0.5 and 2.125 as a fraction form.

Answer: $0.5 = \frac{0.5}{1} = \frac{5}{10} = \frac{1}{2}$

$2.125 = \frac{2.125}{1} = \frac{2125}{1000} = \frac{425}{200} = \frac{85}{40} = \frac{17}{8}$

When we change a rational number $\frac{17}{8}$ into decimal form, one of the following cases will occur

Repeating decimals are decimal numbers in which one or more digits repeat infinitely after the decimal point. If the denominator of a fraction can be expressed as a product of numbers other than 2 and 5, then its decimal form is a repeating decimal. **For example**, the decimal representation of $\frac{1}{3}$ is 0.3333..., where the digit 3 repeats indefinitely.

To represent the repetition of a digit or digits, we use a bar notation above the repeating digit or digits. Thus 0.3333….. = $0.\overline{3}$, 0.23454545454….. = $0.23\overline{45}$

Terminating decimals, on the other hand, are decimal numbers that have a finite number of digits after the decimal point. If the denominator of a fraction can be expressed only as product of numbers 2 and 5, then its decimal form is a terminating decimal. **For instance**, the decimal representation of $\frac{1}{4} = \frac{1}{2*2}$ is 0.25, where there are only two digits after the decimal point.

Converting terminating decimals to fractions

Every terminating decimal can be expressed as a fraction with a denominator that is a power of 10, such as 10, 100, 1000, and so on, depending on the number of decimal places.

Examples : - Convert each of the following decimals to fraction form? a) 7.5 b) - 0.25 c) 4.125

Answer: a) 7.5 = $\frac{7.5}{1} = \frac{75}{10}$ and after simplification, we obtain $\frac{15}{2}$.

b) $-0.25 = -\frac{0.25}{1} = -\frac{25}{100}$ and after simplification, we obtain $-\frac{1}{4}$.

c) $4.125 = \frac{4.125}{1} = \frac{4125}{1000} = \frac{825}{200} = \frac{165}{40} = \frac{33}{8}$ therefore after simplification, we obtain $\frac{33}{8}$.

Representing rational numbers on the number line

Example:- Locate the rational numbers $-2, 3, \frac{3}{5}, -\frac{7}{6}$ on the number line.

Answer: First express fractions in to decimals $\frac{3}{5} = 0.6, and -\frac{7}{6} = -1.1\overline{6}$

2.2.2 Conversion of repeating decimals into fractions

Conversion of repeating decimals into fractions is the process of expressing a decimal number that has a repeating pattern of digits after the decimal point as a fraction in the form of a ratio of two integers.

Example 1: Represent each of the following decimals as a simplest fraction form (ratio of two integers).

a) $0.\overline{5}$ b) $-3.98\overline{6}$

Answer: a) $0.\overline{5}$, let x = $0.\overline{5}$ (after a decimal point there is only one number), so, $10x = $5.\overline{5}$

Then, subtract x from $10x$, $10x - x$ = $5.\overline{5} - 0.\overline{5}$

$9x = 5$

$x = \frac{5}{9}$

Therefore, $0.\overline{5}$ = $\frac{5}{9}$

Answer b) $-3.98\overline{6}$ reject the negative sign and express $3.98\overline{6}$ in to fraction, finally add negative sign is good.

Thus, $3.98\overline{6}$

Let, $x = 3.98\overline{6}$ (the number is written in 3 decimal places), so, $1000x = 3986.\overline{6}$

From 3 decimal places two digits is non - repeating, so $100x = 398.\overline{6}$

Then subtract $100x,$ from $1000x$, $1000x - 100x = 3986.\overline{6} - 398.\overline{6}$

900x = 3588

$x = \frac{3588}{900} = \frac{1196}{300} = \frac{598}{150} = \frac{299}{75}$

Therefore, $3.98\overline{6} = -\frac{299}{75}$